

HOW TO SELL HARD INFORMATION*

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The seller of an asset has the option to buy hard information about the value of the asset from an intermediary. The seller can then disclose this information before selling the asset in a competitive market. We study how the intermediary designs and sells hard information to robustly maximize the intermediary's revenue across all equilibria. Even though the intermediary could use an accurate test that reveals the asset's value, we show that robust revenue maximization leads to a noisy test with a continuum of possible scores. In addition, the intermediary always charges the seller for disclosing the test score to the market, but not necessarily for running the test. This enables the intermediary to robustly appropriate a significant share of the surplus resulting from the asset sale. *JEL Codes:* D82, G24.

I. INTRODUCTION

This article studies settings in which individuals purchase hard information from an intermediary that they can verifiably disclose to influence the actions of others. Such settings are ubiquitous: entrepreneurs often seek evidence that they can disclose to venture capitalists to obtain more funding, sellers of physical and financial assets routinely pay for evaluations that enable them to get better prices, and workers commonly seek certification before applying for positions.

As a concrete example, consider the market for credit ratings. Credit rating agencies rate a variety of financial instruments, from bonds to mortgage-backed securities, as well as the corporations, cities, or countries that issue them. The choice to obtain a credit rating is made by the issuer, and the rating affects the profitability of issuing the financial instrument. Some financial instruments can be issued with or without credit ratings.¹

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1. For example, around 30% of the municipal bonds issued between March 1977 and December 1980 were not rated by either Moody's or Standard and Poor's

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Who pays for credit ratings and at what stage of the rating process? Since the 1970s, the credit rating industry has adopted an “issuer-pays” model, in which the issuer pays for the credit rating. The credit rating agency often provides the issuer with an informal “preliminary credit rating” (or a “shadow rating”), and based on this preliminary rating, the issuer chooses whether to complete the credit rating process. The issuer typically pays for the credit rating when the rating process is completed, but may pay a lower fee if the issuer chooses not to complete the process after the preliminary rating.²

As a result, an issuer can shop for a rating, disclose it if it is favorable, and conceal it if it is unfavorable. According to a Securities and Exchange Commission (SEC) report ([Securities and Exchange Commission 2009](#)), “registrants, among others, can solicit preliminary credit ratings from a rating agency [...] investors are not aware of when registrants seek a preliminary rating or when registrants obtain additional credit ratings but choose not to use them.” The credit rating agency, in turn, may design rating schemes that are attractive to issuers but do not necessarily provide accurate information to investors. “Ratings shopping” and the incentives of credit rating agencies are discussed in a theoretical and empirical literature on credit ratings and disclosure, and in SEC reports (including the aforementioned one) and proposals to amend disclosure rules.³

Our goal is to shed some light on the strategic behavior of information intermediaries—such as credit rating agencies—and those who pay for their services. We consider a stylized game in which the owner of an asset first chooses whether to purchase hard information from a profit-maximizing intermediary, and then

([Reeve and Herring 1986](#)). In 2019, nonrated municipal bonds accounted for more than half of the Bloomberg Barclays benchmark high-yield municipal index; see <https://www.lordabbett.com/en/perspectives/fixedincomeinsights/municipal-bonds-muni-matters-demystifying-nonrated-bonds.html>.

2. Describing the fee structure for credit ratings, an SEC report states that “Typically, the rating agency is paid only if the credit rating is issued, though sometimes it receives a breakup fee for the analytic work undertaken even if the credit rating is not issued” ([Securities and Exchange Commission 2008](#)).

3. [Sangiorgi and Spatt \(2017a\)](#) survey the theoretical literature, which we discuss in detail below. [He, Qian, and Strahan \(2012, 2016\)](#) and [Griffin, Nickerson, and Tang \(2013\)](#) present evidence that the market takes into account both the issuers’ ability to censor unfavorable credit ratings and the incentive of credit rating agencies to cater to issuers.

chooses whether to disclose the information before selling the asset. We focus on the intermediary's design problem: should the intermediary offer accurate or noisy information? What fee structure should be used? How much of the surplus resulting from the sale of the asset can the intermediary extract?

To answer these questions, we abstract from several features of real-world interactions. One such feature is the productive role that an intermediary may play. For example, the information provided by an intermediary may help match less risky products with more risk-averse investors, or alleviate moral hazard or adverse selection. Our analysis highlights how information can be used to generate revenue for the intermediary even without serving a productive role. We also abstract from *ex ante* asymmetric information. This shuts down signaling and focuses our analysis on voluntary disclosure. It is also consistent with some credit rating settings in which a central challenge is not asymmetric information but the difficulty of evaluating the risk associated with the products. An example is a local government that issues a bond and has no particular expertise in evaluating it. Another example is structured products, such as mortgage-backed securities, which are complex and present an evaluation challenge even for credit rating agencies. Accordingly, much of the literature on "ratings shopping" (discussed in the related literature) assumes that all players have symmetric information at the outset.

In our model, an agent owns an asset that he would like to sell in a competitive market. Both the agent and the market have symmetric information about the asset's market value. Before selling the asset, the agent can purchase additional information from an intermediary about the asset's value that he can disclose to the market to improve the terms of trade. The intermediary specifies a test, which stochastically maps the asset's value to a score that can be verifiably disclosed (where a score is the posterior expected value of the asset), a testing fee, and a disclosure fee. If the agent chooses to pay the testing fee, the test is run and he observes the resulting score. He then chooses whether to pay the disclosure fee to disclose the score to the market as hard information. The market cannot distinguish between the agent not disclosing the score and the test not having been run. Because the market is competitive, the market price for the asset following disclosure or nondisclosure equals the asset's expected value conditional on all the information available to the market and the equilibrium choices of the agent.

We show that by choosing a test that generates noisy information and charging a disclosure fee, the intermediary is able to guarantee herself a significant share of the surplus arising from trade even when the equilibrium played by the other parties is chosen adversarially to her interests. This is true even though the information in our model is neither socially valuable nor *ex ante* valuable to the agent. The driving force, as we argue, is the agent's inability to commit not to use the intermediary's services. This implies that the presence of profitable intermediaries in an industry is not, in itself, evidence that the provision of hard information improves the welfare of other market participants. Moreover, even if an intermediary's services generate value, the same profit motive may incentivize her to provide inaccurate information and charge a disclosure fee.

To see how the intermediary can guarantee herself a high revenue in our model, we first observe that the agent's outside option depends on the market's expectation of the agent's behavior. If the market expects the agent to pay the testing fee with some probability but the agent attempts to opt out of the disclosure game by not paying the testing fee, he cannot prove to the market that he chose to opt out. Instead, when the market sees no score being disclosed, it rationally concludes that the test may have been run but the agent chose not to disclose a low score. The market weights this "bad news" contingency and the "no news" contingency in which the agent decided not to pay the testing fee, where the weights depend on what the market believes the agent does in equilibrium. Thus, unlike in a standard mechanism design problem, the agent's outside option depends on both the test-fee structure and the equilibrium played in the induced game between the agent and the market.

If the intermediary can select the equilibrium of the induced game, then an optimal test-fee structure comprises a fully revealing test, a high testing fee, and no disclosure fee. The intermediary selects an equilibrium in which the agent always pays the testing fee, and when the agent chooses not to disclose the score, the market believes the test revealed that the asset has its lowest possible value (say, $\underline{\theta}$). The agent discloses the test score whenever it reveals that the asset's value is not the lowest possible value. The high testing fee charged by the intermediary extracts all of the agent's expected surplus (minus $\underline{\theta}$) from selling the asset, so his expected payoff is $\underline{\theta}$. This is consistent with a key intuition from standard mechanism design: because the agent's payoff beyond

what is needed to satisfy his individual rationality constraint is due to information rents, when the agent starts with no private information, the designer can keep the agent's payoff at his individual rationality level, extract the full surplus, and achieve this by charging an upfront fee.⁴

But the game induced by this test-fee structure has another equilibrium, in which the agent never pays the testing fee so the intermediary's revenue is zero. In this equilibrium, the market treats nondisclosure as “no news,” and the resulting market price is the asset's *ex ante* expected value. Given this, it is optimal for the agent not to pay for the intermediary's services. Thus, choosing this test-fee structure leaves the intermediary vulnerable to obtaining zero revenue. We show in Proposition 1 that this is not an accident: any test-fee structure that has an equilibrium in which the intermediary extracts (approximately) all the surplus also has an equilibrium in which the intermediary's revenue is (approximately) zero.

Motivated by the above discussion, we identify robustly optimal test-fee structures, namely those that guarantee the highest revenue to the intermediary across all equilibria of the induced game. This corresponds to the intermediary choosing the test-fee structure that maximizes her revenue, assuming that the equilibrium of the induced game is selected adversarially to her interests. Our motivation for studying robustly optimal test-fee structures is twofold. First, the intermediary may be unable to coordinate the behavior of the agent and the market on her most preferred equilibrium. The uncertainty about which equilibrium will be played could motivate her to be cautious and therefore use test-fee structures that guarantee her a high revenue across all equilibria.⁵ Second, for any test-fee structure, the sum of the agent and the intermediary's revenue is constant across equilibria and equal to the asset's *ex ante* expected value, so the intermediary's least preferred equilibrium is the agent's most preferred equilibrium. The agent and the market may coordinate on the agent's preferred equilibrium.

4. The intermediary can also extract all the surplus by using a binary score test, making testing free, and charging a high disclosure fee.

5. Our focus on adversarial equilibrium selection is shared by a rapidly growing literature in mechanism and information design, including Bergemann, Brooks, and Morris (2017), Du (2018), Dworzak and Pavan (2020), Halac, Kremer, and Winter (2020), Inostroza and Pavan (2020), Mathevet, Peregó, and Taneva (2020), Ziegler (2020), Halac, Lipnowski, and Rappoport (2021), and Hoshino (2021).

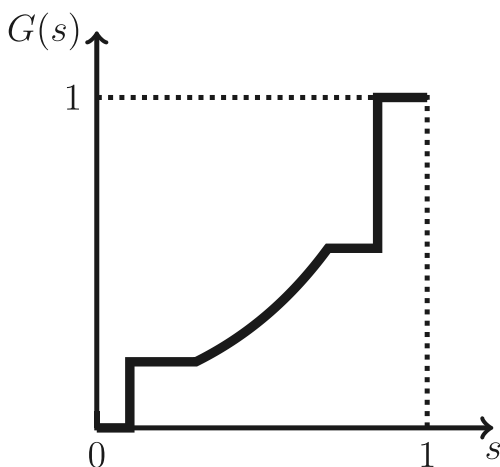


FIGURE I

A Robustly Optimal Score Distribution

A robustly optimal score distribution if the asset quality θ is drawn from $[0,1]$, where G is the marginal CDF on scores. The score distribution features atoms on a low and a high score and an exponential distribution over a continuum of intermediate scores.

Finding the robustly optimal test-fee structure involves optimizing across all test-fee structures while looking at the worst equilibrium in each induced game. Different tests induce different distributions of private information about the asset's value for the agent in the induced games, and different fees change the agent's incentives to obtain and disclose this private information. Thus, the optimization entails comparing the equilibria of disclosure games that vary in both the agent's private information and his disclosure costs. Despite this richness, we find that robustly optimal test-fee structures take a relatively simple form regardless of the distribution of the asset's value. A robustly optimal test generates a marginal distribution of scores that feature an exponential component over an interval of scores (even if the asset's value is drawn from a finite set), one atom below this interval and possibly one atom above it. Such a "step-exponential-step" distribution is illustrated in Figure I. The optimal disclosure fee is always positive, but the optimal testing fee may be positive or zero. The resulting revenue guarantee to the intermediary is positive but bounded away from the full surplus.

To derive these features of the robustly optimal test-fee structure, we first observe that in our model, the intermediary does not provide any added value to the agent *ex ante*. This is because the market draws correct (Bayesian) inferences and is competitive, so for every test-fee structure and any equilibrium, the *ex ante* expected market price is the *ex ante* expected value of the asset. Thus, for every test and for all positive fees, the agent strictly prefers an equilibrium in which the market expects him not to have the asset tested and consequently offers him the asset's *ex ante* expected value. The agent then retains the full surplus and the intermediary's revenue is zero. If the market were to observe whether the asset is tested, the agent could achieve this as an equilibrium outcome by not paying the testing fee. But the market does not observe whether the asset is tested. Therefore, the market's expectation of whether the asset had been tested and the agent observed the resulting score must be consistent with the agent's unobserved equilibrium choice of whether to pay the testing fee.

This is how the intermediary obtains a positive payoff robustly: she uses option value as a carrot to make it noncredible for the agent not to pay for the asset to be tested. Because the market only learns that the test has been run if the agent pays the disclosure fee and discloses the test score, the intermediary creates option value for the agent by offering a test that generates high test scores with some probability and setting sufficiently low testing and disclosure fees. If this option value is sufficiently high, the agent cannot credibly refrain from paying the testing fee. The intermediary then obtains at least the testing fee in every equilibrium. Moreover, the market then treats nondisclosure as concealing a low score, which further motivates the agent to pay the disclosure fee and disclose the test score. The agent is trapped by market expectations that he has paid the testing fee and will disclose if the test score is sufficiently high. This makes the test an irresistible product for the agent.

But even if the agent pays the testing fee with certainty, multiple equilibria may exist. These equilibria differ in the set of scores that the agent discloses and therefore in the probability of disclosure and the intermediary's revenue. The exponential score distribution is robustly optimal because it eliminates potential equilibria in which the agent discloses with low probability. We develop an intuition for this result by showing that the intermediary can be thought of as choosing an optimal "demand curve for

testing,” subject to the demand curve being feasible and the quantity of testing demanded corresponding to the one in the equilibrium least favorable to the intermediary. We illustrate this approach in [Section II](#) and provide a general analysis in [Sections III, IV, and V](#).

[Section VI](#) describes three extensions. First, we show that if the intermediary can charge only a testing fee, a robustly optimal test is binary. We also show that if the intermediary can use only binary tests, it suffices to charge only a testing fee. Second, we consider a setting in which testing is costly for the intermediary. We show that if testing costs increase in the Blackwell order, then our main results continue to hold, that is, there exists a robustly optimal test in the step-exponential-step class. Moreover, if the cost increase is strict, then every robustly optimal test is in this class. Third, we consider an intermediary who can sell the agent multiple pieces of evidence, and gives him the choice of which to disclose. We show that this additional flexibility does not increase the intermediary’s revenue guarantee.

Our work builds on the study of verifiable disclosure and persuasion games, initiated by [Grossman \(1981\)](#) and [Milgrom \(1981\)](#). Their main insight is that if a privately informed agent can costlessly and verifiably disclose evidence about his type, the unique equilibrium involves full disclosure. The subsequent literature suggests a number of mechanisms that dampen this force, including exogenously costly disclosure ([Jovanovic 1982](#); [Verrecchia 1983](#)) and lacking evidence with positive probability ([Dye 1985](#)). [Matthews and Postlewaite \(1985\)](#) and [Shavell \(1994\)](#) consider an uninformed agent who decides whether to take a fully revealing test. [Matthews and Postlewaite \(1985\)](#) show that the unique equilibrium involves testing and full disclosure of the test result when disclosure is voluntary, but involves no testing when disclosure of the test result is mandatory. [Shavell \(1994\)](#) assumes the agent bears a privately known cost of testing and studies how this cost dampens unraveling.

In our model also, the agent faces a cost of obtaining information about her type and a cost of disclosing that information in a verifiable form, and with positive probability, the agent may lack evidence. But we derive these features endogenously because the intermediary chooses the evidence structure and the cost of learning and disclosing the evidence; the probability that the market attributes to the agent having evidence is also determined in equilibrium. Treating these features as endogenous objects reveals a

trade-off: all else equal, the intermediary would like the market to unravel (so that the agent discloses with maximal probability), but the instruments from which she earns revenue are exactly those that counter unraveling. This “quantity-price” trade-off leads to the price-theoretic approach to evidence generation we develop, in which the intermediary chooses the optimal price and designs the optimal demand curve for evidence subject to constraints that correspond to Bayes’s rule and adversarial equilibrium selection.

A closely related strand of the disclosure literature studies choices made by agents to influence market perceptions. Ben-Porath, Dekel, and Lipman (2018) model how an agent chooses projects when he obtains evidence of project returns with positive probability. Their analysis emphasizes option value from the possibility of disclosure as motivating the agent to choose riskier projects. DeMarzo, Kremer, and Skrzypacz (2019) study how an agent chooses tests and disclosures to influence the market valuation of his asset when the choice of test is privately observed. They show that the test and disclosure policy are chosen to minimize the asset’s value conditional on nondisclosure. Shishkin (2019) considers an agent who commits upfront to a test, receives a test score with some exogenous probability, and then chooses whether to disclose the score to a receiver. He shows that a pass/fail test is optimal when the probability of obtaining evidence is low.⁶ In these papers, the agent generates evidence in house, whereas in our work, evidence is generated by a revenue-maximizing intermediary.⁷

Our work complements that of Lizzeri (1999), who is one of the first to study how an intermediary would design and price information. His setting differs from ours in two respects. First, he studies signaling dynamics when the agent is perfectly and privately informed at the outset about the value of the asset. Second, he studies mandatory disclosure, that is, if the agent has the asset tested, he must disclose the score. Mandatory disclosure effectively renders the agent’s testing decision observable to the market. With these features, Lizzeri (1999) shows

6. Pass/fail tests are generally not robustly optimal in our framework, but we show in Section VI.A that if the intermediary can charge only a testing fee, then a binary test is robustly optimal.

7. DeMarzo, Kremer, and Skrzypacz (2019) also consider an extension in which the agent can purchase evidence from a monopolistic intermediary when the equilibria are selected to favor the intermediary.

that the intermediary can extract the full surplus using a nearly uninformative test and a high testing fee.⁸ Our work focuses on voluntary disclosure, that is, when the agent can choose to disclose or conceal the test score. We shut down the signaling channel by studying an agent who is *ex ante* uninformed. We find that once disclosure is voluntary, the intermediary can use option value to induce an *ex ante* uninformed agent to have the asset tested with certainty in every equilibrium. By contrast, were disclosure mandatory (as in [Lizzeri 1999](#)), an uninformed agent could obtain the full surplus by not having the asset tested.

A number of papers have studied related issues in the context of credit rating agencies, with a focus on “ratings shopping” when the issuer pays to disclose credit scores. As the literature has noted, a credit rating agency is then inclined to design rating schemes that induce disclosure. [Skreta and Veldkamp \(2009\)](#) consider the implications of this on investors who are not sufficiently sophisticated to account for selective disclosure. [Mathis, McAndrews, and Rochet \(2009\)](#), [Bolton, Freixas, and Shapiro \(2012\)](#), and [Frenkel \(2015\)](#) offer dynamic reputational models for how credit rating agencies may commit to information structures. [Farhi, Lerner, and Tirole \(2013\)](#) study how an uninformed agent shops for credit when facing a hierarchy of competing intermediaries who have different standards. [Sangiorgi and Spatt \(2017b\)](#) consider equilibrium implications for credit shopping when disclosure is voluntary versus mandatory.

II. AN EXAMPLE

Consider an agent who will sell an asset in a competitive asset market. The market value of the asset, θ , is either 0 or 1, each with equal probability. Neither the agent nor the asset market know the asset's value, but an intermediary can run a test that generates information about the asset's value. The intermediary chooses the test, T , which stochastically maps the asset's value to an unbiased score s in $[0,1]$, so $s = E[\theta|s]$. If the agent wants the asset tested, he has to pay the intermediary a testing fee ϕ_t . If the asset is tested, the score is reported to the agent. The agent then

8. See [Kartik, Lee, and Suen \(2021\)](#) for a variant of Lizzeri's model in which the agent is privately and imperfectly informed about the asset's value. [Harbaugh and Rasmusen \(2018\)](#) study a related model, in which the intermediary's objective is to provide as much information as possible to the receiver.

chooses whether to pay an additional disclosure fee ϕ_d to disclose the score as hard information to the market; otherwise, no score is disclosed to the market. If no score is disclosed, the market cannot distinguish between (i) the asset not being tested and (ii) the asset being tested but the agent not disclosing the score. The intermediary chooses both the test T and the fees (a test-fee structure), and her choice induces a disclosure game between the agent and the asset market. The price the agent obtains for the asset is equal to the expected value of the asset conditional on all the information available to the market and conditional on the equilibrium played. The intermediary's revenue is equal to the fees she collects. Some test-fee structures have multiple equilibria, and the intermediary's objective is to choose a test-fee structure that guarantees her the highest revenue across equilibria.

We use this example to illustrate several features of our analysis. We show that the intermediary benefits from using noisy tests that pool low and high asset values as intermediate scores. We depict how our problem maps to setting a price on an optimally chosen demand curve, and we show why the robustly optimal test uses an exponential score distribution. To illustrate these features, it suffices to assume that the intermediary can charge only a disclosure fee ϕ_d (so testing is free). As we later show, these features also arise when it is optimal to charge a testing fee.

We first describe the intermediary's revenue guarantee if she uses a fully revealing test, that is, a test in which the score is equal to the asset's value θ . [Figure II](#), Panel (a) depicts our analysis of the fully revealing test using a (inverse) demand correspondence. The demand correspondence (the solid black curve) traces for any disclosure fee in $[0,1]$ (on the vertical axis) the corresponding probabilities of disclosure consistent with the equilibria of the induced game (on the horizontal axis). We now describe this set of equilibria.

For any disclosure fee in $[0,1]$, there is an equilibrium in which the agent has the asset tested, conceals a score of 0, and discloses a score of 1. Because the asset's score is 1 with probability $\frac{1}{2}$, the disclosure probability is $\frac{1}{2}$. Segment C in [Figure II](#), Panel (a) depicts a disclosure probability of $\frac{1}{2}$ for every disclosure fee in $[0,1]$. To see why concealing a score of 0 and disclosing a score of 1 is an equilibrium, observe that if the market believes that the agent is using this strategy, then the market concludes that the agent obtained a score of 0 if no score is disclosed. Therefore, if no score is disclosed, the market offers a price of 0 for the asset. Given

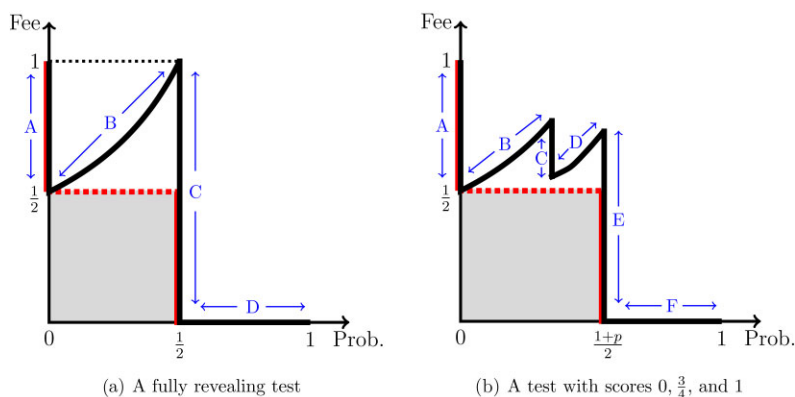


FIGURE II

Demand Correspondences

The solid black curves are the demand correspondences, and the dashed red curves (color version online) are the robust demand curves.

this market price, the agent has no incentive to deviate: the payoff of $1 - \phi_d$ from disclosing a score of 1 weakly exceeds the payoff from not disclosing, which is weakly higher than the payoff from disclosing a score of 0. In this equilibrium, the resulting revenue for the intermediary is half the disclosure fee. Thus, by charging a disclosure fee of 1, the intermediary extracts the entire expected market value of the asset. This equilibrium with a disclosure fee of 1 corresponds to the highest point of segment C.

As we see in Figure II, the equilibrium described above is unique when the disclosure fee is in $(0, \frac{1}{2})$.⁹ But for disclosure fees that weakly exceed $\frac{1}{2}$, other equilibria exist. There is an equilibrium in which the agent has the asset tested but never discloses the score, so the disclosure probability is 0. Segment A in Figure II depicts these equilibria. Because the market does not expect any score to be disclosed, it does not draw any inferences about the asset's value from nondisclosure. Therefore, if no score is disclosed, the market offers a price of $\frac{1}{2}$, which is the ex ante expected market value of the asset. Given this market price, the agent prefers not to disclose a score of 1 (or 0) because the disclosure fee is at least $\frac{1}{2}$. Because the agent does not disclose any score, the

9. If the disclosure fee is 0, then for any probability p , there is an equilibrium in which the agent discloses a score of 1 and with probability p discloses a score of 0. Segment D in Figure II, Panel (a) depicts these equilibria.

TABLE I
DISTRIBUTION OF SCORE s CONDITIONAL ON ASSET VALUE θ , WHERE p IS IN $(0, \frac{1}{3})$

	$s = 0$	$s = \frac{3}{4}$	$s = 1$
$\theta = 0$	$1 - p$	p	0
$\theta = 1$	0	$3p$	$1 - 3p$

intermediary's revenue is 0 in this equilibrium. Thus, a disclosure fee of 1 leads to a revenue of 0 in this equilibrium. This contrasts with the equilibrium described in the previous paragraph where a disclosure fee of 1 leads to a revenue equal to the entire expected market value of the asset.

The demand correspondence also shows a mixed-strategy equilibrium in which the agent conceals a score of 0 and discloses a score of 1 with an interior probability. Segment B in Figure II, Panel (a) depicts these equilibria. As segment B makes clear, a higher disclosure fee is associated with a higher probability of disclosure. The reason is that increasing the disclosure probability reduces the market price following nondisclosure, which makes the agent willing to pay a higher disclosure fee.

Since we study the intermediary's revenue guarantee across equilibria, we identify the "robust demand curve" for disclosure, which maps each disclosure fee (price) to the lowest probability of disclosure (quantity) across all the equilibria associated with that fee. This is the dashed red curve in Figure II, Panel (a). The revenue guarantee for a given disclosure fee is the product of the disclosure fee and the lowest equilibrium probability of disclosure, that is, the area of the rectangle under the robust demand curve at the point associated with that disclosure fee. The maximal revenue guarantee of $\approx \frac{1}{4}$ for the fully revealing test is the area of the shaded rectangle in Figure II, obtained from a disclosure fee that is slightly below $\frac{1}{2}$.¹⁰

Let us now see how the intermediary can improve her robust guarantee by introducing an intermediate score that pools the asset's two possible values, 0 and 1. Consider a test with three possible scores—0, $\frac{3}{4}$, and 1—generated by the conditional distribution in Table I. The demand correspondence and the robust demand

10. As is common with adversarial equilibrium selection, the intermediary cannot robustly achieve a payoff of exactly $\frac{1}{4}$ with a disclosure fee of $\frac{1}{2}$, since there is then an equilibrium in which she obtains a payoff of 0.

curve for this test are shown in Figure II, Panel (b). For strictly positive disclosure fees less than $\frac{1}{2}$, and if p is sufficiently small, there is a unique equilibrium: the agent has the asset tested, and discloses his score if it is $\frac{3}{4}$ or 1; if no score is disclosed, the market offers a price of 0. The probability of disclosure in this equilibrium is $\frac{1+p}{2}$. Segment *E* in Figure II, Panel (b) depicts these equilibria.¹¹ For disclosure fees higher than $\frac{1}{2}$, other equilibria exist, and all of them feature a lower probability of disclosure.¹²

Thus, by setting a disclosure fee slightly below $\frac{1}{2}$, the intermediary obtains a revenue of $\frac{1+p}{4}$, which is higher than her revenue guarantee of $\frac{1}{4}$ from the robustly optimal fully revealing test. By introducing the intermediate score, the intermediary increases the quantity of disclosure demanded in the worst equilibrium because the agent pays the disclosure fee not only when the asset's value is 1 but also, with some probability, when its value is 0.

The intermediary is limited, however, in how much pooling she can introduce: a value of p that is too high introduces a bad equilibrium in which the agent has the asset tested but only discloses a score of 1.¹³ If no score is disclosed, the market concludes that the score is either 0 or $\frac{3}{4}$ and offers a price of $\frac{3p}{1+3p}$. In this equilibrium, the probability of disclosure is $\frac{1-3p}{2}$, leading to a revenue of $\frac{1-3p}{4} < \frac{1}{4}$. To rule out this equilibrium, the intermediary sets the value of p so that if the score is $\frac{3}{4}$, the agent prefers to disclose his score rather than obtain the nondisclosure price:

$$\underbrace{\frac{3}{4} - \phi_d}_{\text{Disclose a score of } \frac{3}{4}} > \underbrace{\frac{3p}{1+3p}}_{\text{Nondisclosure price}} .$$

11. If the disclosure fee is 0, then for any probability p , there is an equilibrium in which the agent discloses scores $\frac{3}{4}$ and 1, and with probability p discloses a score of 0. Segment *F* in Figure II, Panel (b) depicts these equilibria.

12. Segment *A* in Figure II, Panel (b) corresponds to equilibria in which the agent has the asset tested but never discloses the score; segment *B* corresponds to equilibria in which the agent conceals scores 0 and $\frac{3}{4}$, and discloses a score of 1 with an interior probability; segment *C* corresponds to equilibria in which the agent conceals scores 0 and $\frac{3}{4}$ and always discloses a score of 1; segment *D* corresponds to equilibria in which the agent conceals a score of 0, discloses a score of 1, and discloses a score of $\frac{3}{4}$ with an interior probability.

13. This corresponds to the lowest point of segment *C* in Figure II, Panel (b) moving below the dashed red line, which changes the robust demand curve and hurts the intermediary.

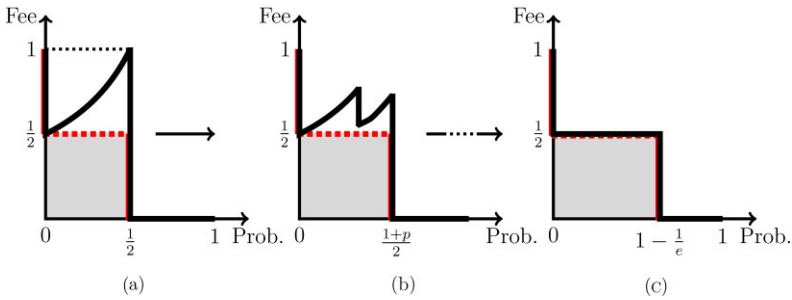


FIGURE III

Flattening the Demand Correspondence

Adding score $\frac{3}{4}$ to the fully revealing test, thus transitioning from Panel (a) to Panel (b), flattens the demand correspondence (in solid black) and pushes the robust demand curve (in dashed red) to the right. Additional scores can be added to further flatten the demand correspondence and push the robust demand curve to the right. This ultimately leads to the optimal test (Panel (c)) with a continuum of scores, for which the demand correspondence (in solid black) is flat and coincides with the robust demand curve (in dashed red).

With a disclosure fee slightly below $\frac{1}{2}$, the above inequality holds for p less than $\frac{1}{9}$. The revenue guarantee is attained by setting $p = \frac{1}{9}$ and charging a disclosure fee slightly below $\frac{1}{2}$. This restriction on p is a “cross-equilibrium” constraint, which ensures that a low-revenue equilibrium is not created. Cross-equilibrium constraints lead to the exponential score distribution in the robustly optimal tests.

To see why, let us return to the demand curve approach. Adding the score $\frac{3}{4}$ to the fully revealing test “flattens” the demand curve, which pushes the robust demand curve to the right, as illustrated in the transition from Figure III, Panel (a) to Figure III, Panel (b). Additional scores further flatten the demand curve and push the robust demand curve to the right. The degree to which this can be done is limited by the cross-equilibrium constraints, and this leads to tests that induce a rectangular demand correspondence like the one in Figure III, Panel (c): intuitively, whenever the demand correspondence or the robust demand curve are not a rectangle, there is some slack that the intermediary can use to change the test in a way that increases the probability of disclosure (by having more pooling) in the equilibrium that has the lowest probability of disclosure, that is, without violating cross-equilibrium constraints.

Which tests lead to rectangular demand correspondences? Those whose marginal score distribution includes an exponentially distributed component. To see why, notice that with a rectangular demand correspondence, there is a disclosure fee ϕ_d for which there is an interval of equilibrium disclosure probabilities. In [Figure III](#), Panel (c), this disclosure fee is $\phi_d = \frac{1}{2}$. This continuum of equilibria implies that there is a continuum of scores s for which

$$(1) \quad s - \phi_d = E[s' | s' \leq s].$$

The left side above is the payoff from disclosing a score s , and the right side is that from not disclosing when the set of scores that do not disclose are those weakly below s . Thus, when equation (1) holds, there is an equilibrium with a disclosure threshold of s . Using $G(s)$ as the marginal CDF on scores and $G(s' | s' \leq s)$ as the conditional CDF on scores no higher than s , the right side can be rewritten as $\int_0^s (1 - G(s' | s' \leq s)) ds'$. It follows then that equation (1) is equivalent to

$$(2) \quad \phi_d = \int_0^s G(s' | s' \leq s) ds' = \frac{\int_0^s G(s') ds'}{G(s)} = \left(\frac{d}{ds} \ln \left(\int_0^s G(s') ds' \right) \right)^{-1}.$$

Because the above is true for an interval of scores s , it defines a differential equation whose solution is $G(s) = \alpha e^{\frac{s}{\phi_d}}$ for some constant α , that is, the exponential score distribution.¹⁴

We illustrate this solution in [Figure IV](#), where Panel (a) shows the marginal score distribution in the robustly optimal test. This marginal distribution is generated by a test in which if the value of the asset is 0, with probability $\frac{2}{e}$ the test score is 0 and with

14. The exponential score distribution may bring to mind the work of [Roesler and Szentes \(2017\)](#) and [Condorelli and Szentes \(2020\)](#). In those papers and in our work, a player optimally chooses a demand curve by manipulating an information structure or a distribution of values. In those papers, a buyer optimally chooses a unit-elastic demand curve that leads a seller to charge a low price and be indifferent between that price and higher prices. In a similar vein, [Ortner and Chassang \(2018\)](#) show that a principal can reduce the cost of monitoring by paying a monitor wages that generate a unit-elastic demand curve for bribes. In our work, adversarial equilibrium selection leads the intermediary to choose a test that generates a rectangular demand curve, which corresponds to an exponential distribution of scores. The driving force is not to generate indifference but to eliminate equilibria in which the agent does not disclose intermediate scores.

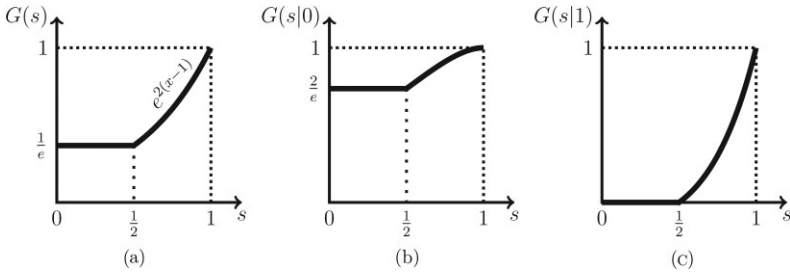


FIGURE IV

The Robustly Optimal Distribution of Scores

The robustly optimal distribution of scores where (Panel (a)) $G(\cdot)$ is the marginal probability, (Panel (b)) $G(\cdot|0)$ is the conditional probability of scores for $\theta = 0$, and (Panel (c)) $G(\cdot|1)$ is the conditional probability for $\theta = 1$.

complementary probability the test score is distributed exponentially on $[\frac{1}{2}, 1]$, as shown in Figure IV, Panel (b), and if the value of the asset is 1, the test score is distributed exponentially on $[\frac{1}{2}, 1]$, as shown in Figure IV, Panel (c). The testing fee is 0 (by assumption), and the disclosure fee is slightly less than $\frac{1}{2}$. This test-fee structure induces a game with a unique equilibrium: the agent has the asset tested and discloses the score if it exceeds the disclosure fee. This leads to partial pooling: if the asset value is low, then with some probability the test generates scores that are disclosed and coincide with those generated if the asset value is high. If no score is disclosed the market offers a price of 0. The resulting revenue for the intermediary is $\approx \frac{1}{2}(1 - \frac{1}{e})$, which is more than $\frac{1}{4}$ but less than the full surplus of $\frac{1}{2}$.

III. MODEL

III.A. The Setting

A risk-neutral agent sells an asset in a competitive market comprising two risk-neutral buyers who have a common value for the asset. The agent and both buyers are initially symmetrically informed about the asset's market value, θ , knowing only that it is drawn according to a distribution F with support $\Theta \subseteq [\underline{\theta}, \bar{\theta}]$, where $0 \leq \underline{\theta} < \bar{\theta} < \infty$ are the lowest and highest values in Θ , with an expected value of μ .

Prior to selling the asset, the agent can pay an intermediary to evaluate the asset and has the option, for an additional fee, to

disclose that evaluation to the market. We refer to the evaluation scheme and the fees as a test-fee structure. A test-fee structure comprises:

- i. a test, T , that stochastically maps the asset's value to a score in some set S ,¹⁵
- ii. a testing fee, $\phi_t \in \mathbb{R}$, which the agent pays for the asset to be evaluated; and
- iii. a disclosure fee, $\phi_d \in \mathbb{R}$, which the agent pays to disclose the score to the market.

Because the test will only be used to determine the asset's expected value, we henceforth assume without loss of generality that each test score is an unbiased estimate of the value. That is, $S \subseteq [\underline{\theta}, \bar{\theta}]$ and for every score s in S , $E[\theta|s] = s$. We denote the set of all tests by \mathcal{T} . For each test T , let $G_T(s)$ denote the probability that if the intermediary evaluates the asset using test T , the resulting score is at most s . We refer to G_T as the marginal score distribution.

Turning to fees, we denote the pair of testing and disclosure fees by $\phi \equiv (\phi_t, \phi_d)$. The testing fee is the price the intermediary charges to generate information about the asset. The disclosure fee is the additional price to make that information hard or verifiable so that the agent can disclose it to the buyers.

We denote a test-fee structure by (T, ϕ) . A test-fee structure (T, ϕ) induces a game $\mathcal{G}(T, \phi)$ between the agent and the buyers, in which (T, ϕ) is common knowledge. The game has the following timeline:

- i. Nature chooses the value θ of the asset according to the distribution F .
- ii. The agent, without observing θ , decides whether to have the asset tested. If so, he pays the testing fee ϕ_t and observes a score s drawn according to T .
- iii. The agent decides whether to disclose the score to the market. If so, he pays the disclosure fee ϕ_d , and the buyers observe the score. If he does not disclose, or if he does not

15. Formally, S is a Polish space and the test $T : \mathcal{B}(S) \times \Theta \rightarrow [0, 1]$ is a Markov kernel, where $\mathcal{B}(S)$ is the Borel σ -algebra on S . The interpretation is that if the asset value is θ the test generates a random score according to $T(\cdot, \theta)$. Distribution F and test T induce a joint distribution over $\Theta \times S$.

- have the asset tested, the buyers observe a null message, N .
- iv. The buyers bid simultaneously for the asset, and the asset is sold to the highest bidder, with ties broken uniformly.

The agent's payoff is the price at which he sells the asset minus any testing and disclosure fees that he pays. The payoff of the buyer that buys the asset at price p is $\theta - p$; the other buyer's payoff is 0.¹⁶

In the induced game $\mathcal{G}(T, \phi)$, an agent's (behavioral) strategy is (σ_t, σ_d) , where σ_t in $[0,1]$ is the probability with which he has the asset tested and $\sigma_d: S \rightarrow [0, 1]$ is a measurable function that specifies the score-contingent probability with which he discloses the score. Buyer i 's (pure) strategy $\sigma_i: S \cup \{N\} \rightarrow \mathbb{R}$ specifies buyer i 's bid following a disclosure of score s or the null message N . A belief system ρ_i specifies buyer i 's posterior belief about the asset's value following a disclosure of score s or the null message N . We denote a strategy profile $\sigma = (\sigma_t, \sigma_d, \sigma_1, \sigma_2)$ and belief system $\rho = (\rho_1, \rho_2)$ by (σ, ρ) .

For a game $\mathcal{G}(T, \phi)$, we denote the set of perfect Bayesian equilibria (PBE) by $\Sigma(T, \phi)$.¹⁷ Because the buyers have the same beliefs and compete in a first-price auction, each buyer's bid is equal to the asset's expected value given all the available information.

III.B. Maximal Revenue Guarantees

We study the intermediary's maximal revenue guarantee, namely, the highest payoff that she can guarantee herself by choosing a test-fee structure, that is, assuming that the equilibrium in the induced game is chosen adversarially to her interests.

16. We have framed our analysis in terms of this market game for concreteness. But the issues and our analysis apply to other settings in which an agent obtains evidence to persuade others. For example, a principal may decide how much to invest in a project based on her beliefs about the project's success, and the agent may acquire evidence to persuade the principal to invest more. Our model is isomorphic to such a setting if the principal's investment increases linearly in her posterior expectation and the agent's payoff increases linearly in the principal's investment.

17. Specifically each player behaves in a way that is sequentially rational, beliefs μ_i are derived via Bayes's rule whenever possible, and off-path beliefs satisfy "You can't signal what you don't know" (Fudenberg and Tirole 1991). The important implication for our setting is that both on and off the equilibrium path, each buyer's posterior expected value after the agent discloses score s is s .

DEFINITION 1. The intermediary's maximal revenue guarantee is

$$(3) \quad R_M \equiv \sup_{(T, \phi) \in T \times \mathbb{R}^2} \inf_{(\sigma, \rho) \in \Sigma(T, \phi)} \sigma_t \left(\phi_t + \phi_d \int_S \sigma_d(s) dG_T \right).$$

Recall that σ_t is the probability with which the agent pays for the intermediary to test the asset and $\sigma_d(s)$ is the probability with which the agent pays to disclose a score of s . The first term in the parenthesis in equation (3) is the intermediary's revenue from the testing fee, and the second term is the revenue from the disclosure fee.

Why do we study the maximal revenue guarantee? One reason is that any test-fee structure that has an equilibrium in which the intermediary's revenue is high also has an equilibrium in which her revenue is low. To formalize this, consider the full informational surplus $R_F \equiv \mu - \underline{\theta}$, where μ is the expected value of the asset. The full informational surplus is a tight upper bound on the intermediary's revenue in any equilibrium.¹⁸ But any test-fee structure that has an equilibrium in which the intermediary obtains the full informational surplus also has an equilibrium in which the intermediary obtains 0. We alluded to this fact in the introduction and in the example in Section II. In fact, for any test-fee structure in which the intermediary obtains close to the full informational surplus in some equilibrium, there is another equilibrium in which the intermediary obtains close to 0.

PROPOSITION 1. There exists $\delta > 0$ such that for any small $\varepsilon \geq 0$, any test-fee structure that has an equilibrium in which the intermediary's revenue is at least $R_F - \varepsilon$ also has another equilibrium in which the intermediary's revenue is at most $\delta\sqrt{\varepsilon}$.

Proposition 1 formalizes the challenge that multiple equilibria present to the intermediary: choosing a test-fee structure because its most favorable equilibrium generates a high revenue leaves the intermediary vulnerable to an equilibrium that generates low or zero revenue. An immediate implication of Proposition 1 is that the maximal revenue guarantee R_M is bounded away from

18. The intermediary can extract the full informational surplus using a fully revealing test and an appropriate testing fee. But she cannot extract more than this because the total surplus in the economy is μ and the market price of the asset is never lower than $\underline{\theta}$.

the full information surplus R_F . Our main result, Proposition 2, derives a tight bound on R_M that, for every μ , applies across distributions for which the asset's expected value is μ . Because the proof of Proposition 1 uses techniques that we develop later in the article, we do not develop the intuition here. The proof and a graphical intuition are in the [Online Appendix](#).

A separate rationale for focusing on the maximal revenue guarantee is that for any test-fee structure, an equilibrium that minimizes the intermediary's revenue also maximizes the agent's payoff.¹⁹ Thus, if the intermediary worries that the agent and the asset market will coordinate on the agent's preferred equilibrium, she would choose a test-fee structure that attains her maximal revenue guarantee.

Our analysis also shows how to maximize the intermediary's revenue across test-fee structures that admit a unique equilibrium. This maximal revenue is clearly at most R_M , but because the robustly optimal test-fee structure that we identify in Proposition 2 has a unique equilibrium, it is also a solution to this related problem.

IV. PRELIMINARY STEPS

To simplify the problem of solving for the maximal revenue guarantee, we take the following preliminary steps:

- i. We frame the analysis of tests purely in terms of marginal score distributions.
- ii. We show that it suffices to consider only those test-fee structures in which the agent pays the testing fee with probability 1 in every equilibrium.
- iii. For every such test-fee structure, we characterize the equilibrium with the lowest revenue for the intermediary in terms of a score threshold for disclosure.
- iv. Because finding the maximal revenue guarantee involves optimizing with strict constraints, we formulate a relaxed problem with weak constraints that has the same solution.

19. Notice that the equilibria that maximize the agent's payoff are the only Pareto-efficient ones from the perspective of the players in the game induced by the test-fee structure.

IV.A. *Shifting from Tests to Marginal Score Distributions*

For the intermediary's revenue, all that matters about a test is its marginal score distribution: for all fees, if two tests have the same marginal score distribution, then they have the same equilibria.²⁰ Thus, henceforth, we refer to (G, ϕ) as a test-fee structure, where G is a CDF on $[\underline{\theta}, \bar{\theta}]$ that corresponds to the marginal score distribution of some test T and ϕ is a pair of testing and disclosure fees.

Focusing on the set of marginal score distributions induced by all possible tests is useful because this set is easy to characterize. Recall that a distribution G is a mean-preserving contraction of F if its support is in $[\underline{\theta}, \bar{\theta}]$ and $\int_{\underline{\theta}}^{s'} G(s)ds \leq \int_{\underline{\theta}}^{s'} F(s)ds$ for all $s' \in [\underline{\theta}, \bar{\theta}]$, with equality at $s' = \bar{\theta}$. We denote by $\Gamma(F)$ the set of distributions that are mean-preserving contractions of F . We then have the following classical result (Blackwell 1953; Strassen 1965).

LEMMA 0. A marginal score distribution G is induced by some (unbiased) test if and only if G is in $\Gamma(F)$.

We use this formulation to rewrite revenue guarantees: for a test-fee structure (G, ϕ) , let $\hat{\Sigma}(G, \phi)$ be the equilibrium set of the induced game between the agent and the market. The revenue guarantee of (G, ϕ) is the lowest revenue generated across the equilibria of this test-fee structure: $R(G, \phi) \equiv \inf_{(\sigma, \rho) \in \hat{\Sigma}(G, \phi)} \sigma_t(\phi_t + \phi_d \int_S \sigma_d(s) dG)$. The maximal revenue guarantee is thus $R_M = \sup_{(G, \phi) \in \Gamma(F) \times \mathbb{R}^2} R(G, \phi)$.

IV.B. *Using Option Value as a Carrot*

We show that in every test-fee structure, either the asset is tested with probability 1 in every equilibrium or there exists an equilibrium in which the asset is tested with probability 0 (and the intermediary's revenue is 0).

20. Take two test-fee structures (T, ϕ) and (T', ϕ) such that T and T' have the same marginal score distribution. Take an equilibrium (σ, ρ) of $\mathcal{G}(T, \phi)$. Suppose that (σ, ρ) were played in $\mathcal{G}(T', \phi)$ and observe that given ρ and that $G_T = G_{T'}$, the strategy profile σ remains sequentially rational for the agent and the buyers. Moreover, (σ, ρ) continues to satisfy Bayes's rule and the appropriate consistency condition. Therefore, (σ, ρ) is an equilibrium of $\mathcal{G}(T', \phi)$.

LEMMA 1. If a test-fee structure (G, ϕ) satisfies

$$(P) \quad \phi_t < \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG,$$

then the asset is tested with probability 1 in every equilibrium; otherwise, there exists an equilibrium in which the asset is tested with probability 0.

The logic of Lemma 1 is that (P) is a participation constraint that must hold for the asset to be tested with positive probability in every equilibrium. To see why, consider a test-fee structure (G, ϕ) and an equilibrium in which the asset is tested with probability 0. Because the market expects nondisclosure with probability 1, the price of the asset conditional on nondisclosure is μ . If the agent deviates and has the asset tested, he optimally pays ϕ_d to disclose the score whenever it is higher than $\mu + \phi_d$. This deviation is strictly profitable if

$$\mu < -\phi_t + \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} \max\{\mu, s - \phi_d\} dG}_{\text{Option Value}},$$

which, by rearranging, yields (P). Thus, the asset is tested with positive probability in every equilibrium if and only if (P) holds. The proof of Lemma 1 shows that if (P) holds, the asset is in fact tested with probability 1 in every equilibrium.

This result also allows us to show that it suffices to restrict attention to nonnegative fees: for any test-fee structure with a negative testing or disclosure fee, the intermediary can improve her revenue guarantee by using a test-fee structure that has nonnegative fees.

LEMMA 2. For any test-fee structure (G, ϕ) with $\phi_d < 0$ or $\phi_t < 0$, there exists a test-fee structure (G', ϕ') with nonnegative fees such that $R(G, \phi) < R(G', \phi')$.

IV.C. Adversarial Disclosure Thresholds

For any test-fee structure that satisfies (P), an adversarial equilibrium minimizes the disclosure probability while having the asset tested with probability 1. We show that such equilibria are

threshold equilibria in which the agent discloses the score if it strictly exceeds the threshold.

With Lemmas 1 and 2 in hand, we restrict attention to test-fee structures that satisfy (P) and have nonnegative disclosure fees. Given a test-fee structure (G, ϕ) , consider thresholds τ that weakly exceed s_G , where s_G is the lowest score in the support of G . We say that such a threshold τ is an equilibrium threshold for a test-fee structure (G, ϕ) if

$$(4) \quad \tau - \phi_d = E_G[s | s \leq \tau].$$

If τ satisfies equation (4), then there is an equilibrium in which the agent discloses the score if and only if it strictly exceeds τ . To see why, suppose that the agent behaves in this way and equation (4) holds. Then the left side of equation (4) is the difference between the market price of the asset when a score of τ is disclosed and the disclosure fee, and the right side is the market price for the asset when no score is disclosed. Thus, the agent is indifferent between disclosing and not disclosing a score of τ . Because the market price following disclosure increases in the score but the market price following nondisclosure is constant in the score, the agent strictly prefers not to disclose scores lower than τ and to disclose scores higher than τ .

A test-fee structure may have multiple equilibrium thresholds. It may also have other equilibria, in which if the agent obtains a threshold score, he chooses to disclose his score with strictly positive probability rather than probability 0. The mixed strategy equilibria for the fully revealing and the three-score tests in Sections II are examples of such equilibria. We prove in Lemma 3 that from the standpoint of adversarial equilibrium selection, it suffices to focus on equilibria in which the agent discloses his score only if it strictly exceeds the threshold.

We proceed as follows. For each test-fee structure, we show that a highest equilibrium threshold exists and provide a characterization of the highest equilibrium threshold that we later use to find the robustly optimal test-fee structure. We then show that this highest equilibrium threshold corresponds to an adversarial equilibrium in which the agent discloses his score if and only if it strictly exceeds this threshold.

LEMMA 3. If a test-fee structure (G, ϕ) satisfies (P) and $\phi_d \geq 0$, then the following are true:

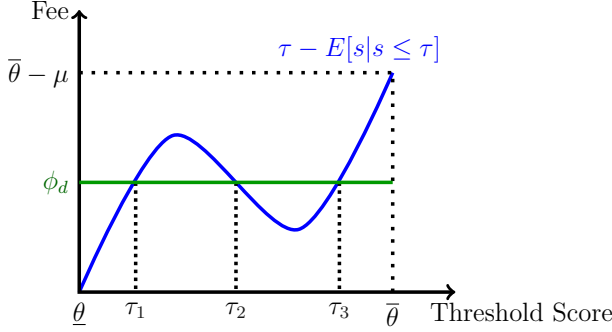


FIGURE V

Characterizing the Highest Equilibrium Threshold

The adversarial equilibrium for a given disclosure fee is the equilibrium with the highest equilibrium threshold. For example, with a disclosure fee of ϕ_d , the highest equilibrium threshold is τ_3 .

- i. A highest equilibrium threshold τ exists.
- ii. A score threshold $\tau \geq \underline{s}_G$ is the highest equilibrium threshold if and only if

$$\begin{aligned}
 \text{(HE)} \quad & \tau - \phi_d = E_G[s|s \leq \tau], \\
 & \tau' - \phi_d > E_G[s|s \leq \tau'] \quad \forall \tau' > \tau.
 \end{aligned}$$

- iii. There exists an adversarial equilibrium in which the agent discloses score s if and only if $s > \tau$, where τ is the highest equilibrium threshold.

Figure V illustrates our characterization of the highest equilibrium threshold.

IV.D. A Relaxed Problem with an Identical Value

We have seen that if a test-fee structure satisfies (P), then the agent takes the test with probability 1 and the adversarial equilibrium is characterized by the highest equilibrium threshold. The revenue guarantee in the robustly optimal test-fee structure is therefore

$$R_M = \sup_{(G, \phi, \tau) \in \Gamma(F) \times \mathbb{R}^3} \phi_t + \phi_d(1 - G(\tau)) \quad \text{s.t. (P) and (HE)}$$

The intermediary's maximization problem has constraints with strict inequalities, but we show that the problem's value R_M is unchanged if those inequalities are made weak. These are the weak participation constraint,

$$(w-P) \quad \phi_t \leq \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG$$

and the weak-highest equilibrium constraint (which defines the weak-highest equilibrium threshold),

$$(w-HE) \quad \begin{aligned} \tau - \phi_d &= E_G[s | s \leq \tau] \\ \tau' - \phi_d &\geq E_G[s | s \leq \tau'] \quad \forall \tau' > \tau. \end{aligned}$$

We write the relaxed problem as

$$\max_{(G, \phi, \tau) \in \Gamma(F) \times \mathbb{R}^3} \phi_t + \phi_d(1 - G(\tau)) \quad \text{s.t. (w-P) and (w-HE).}$$

In addition to having a solution and the same value as the original problem, the relaxed problem has several attractive features. First, for any test-fee structure that solves the relaxed problem, there is a “nearby” test-fee structure whose revenue guarantee is close to the maximal revenue guarantee R_M . Second, for any convergent sequence of test-fee structures that achieves the maximal revenue guarantee, the limiting test-fee structure is a solution to the relaxed problem. Thus, solutions to the relaxed problem identify necessary and sufficient features of test-fee structures whose revenue guarantees approximate R_M . We call these solutions robustly optimal test-fee structures.

Let us formalize this discussion. A sequence of test-fee structures $\{(G^n, \phi^n)\}_{n=1, 2, \dots}$ converges to a test-fee structure (G, ϕ) if G^n converges weakly to G and ϕ^n converges to ϕ . For a test-fee structure (G, ϕ) and equilibrium threshold τ , we denote the associated revenue for the intermediary by $\hat{R}(G, \phi, \tau)$. We use this notation to link the relaxed and original problems.

LEMMA 4.

- i. An optimal solution (G, ϕ, τ) to the relaxed problem exists and $R_M = \hat{R}(G, \phi, \tau)$.
- ii. Consider any optimal solution (G, ϕ, τ) to the relaxed problem. Then there exists a sequence of test-fee structures and thresholds $\{(G^n, \phi^n, \tau^n)\}_{n=1, 2, \dots}$ such that (a)

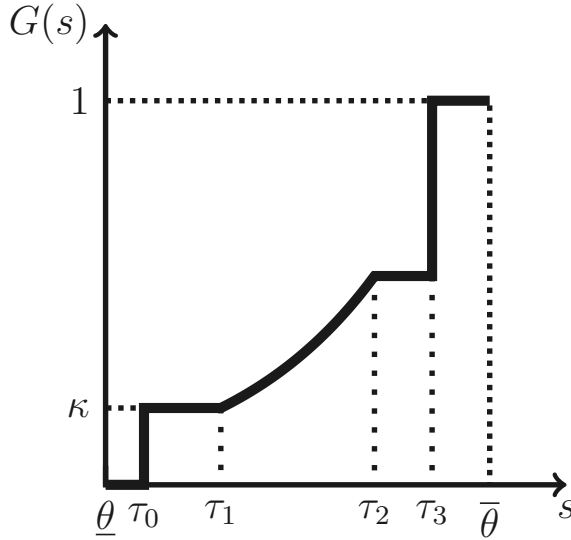


FIGURE VI

A Step-Exponential-Step Distribution G

- for each n , (G^n, ϕ^n, τ^n) satisfies (P) and (HE), (b) (G^n, ϕ^n) converges to (G, ϕ) , and (c) $R_M = \lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n)$.
- iii. Consider any sequence of test-fee structures and thresholds $\{(G^n, \phi^n, \tau^n)\}_{n=1, 2, \dots}$ such that (a) for each n , (G^n, ϕ^n, τ^n) satisfies (P) and (HE), (b) (G^n, ϕ^n) converges to a test-fee structure (G, ϕ) , and (c) $R_M = \lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n)$. Then there exists τ such that (G, ϕ, τ) satisfies (w-P) and (w-HE), and $R_M = \hat{R}(G, \phi, \tau)$.

V. ROBUSTLY OPTIMAL TEST-FEE STRUCTURES

This section contains our main result. We show that robustly optimal test-fee structures use tests that have a “step-exponential-step” form; that is, the distributions over scores are exponential over an interval, have up to two mass points, one above and one below the interval, and have zero density everywhere else. We also show that the optimal disclosure fees are strictly positive, and we derive a tight bound on the intermediary’s maximal revenue guarantee.

As illustrated in Figure VI, a test-fee structure (G, ϕ) is in the step-exponential-step class if there exists $\kappa \in [0, 1]$ and thresholds

$\tau_0 < \tau_1 < \tau_2 \leq \tau_3$ such that

$$(5) \quad G(s) = \begin{cases} \kappa & \text{if } s \in [\tau_0, \tau_1) \\ \kappa e^{\frac{s-\tau_1}{\tau_1-\tau_0}} & \text{if } s \in [\tau_1, \tau_2] \\ 1 & \text{if } s \geq \tau_3, \end{cases}$$

G assigns probability 0 to $[0, \tau_0) \cup (\tau_2, \tau_3)$, and the fees are

$$(6) \quad \phi_d = \tau_1 - \tau_0,$$

$$(7) \quad \phi_t = \left(1 - \kappa e^{\frac{\tau_2-\tau_1}{\phi_d}}\right)(\tau_3 - (\mu + \phi_d)).$$

PROPOSITION 2. For any distribution F of the asset's value, the following hold:

- i. There exists a test-fee structure in the step-exponential-step class that is robustly optimal.
- ii. Every robustly optimal test-fee structure has a strictly positive disclosure fee.
- iii. If (G, ϕ) is a robustly optimal test-fee structure, then the testing fee ϕ_t is strictly positive if and only if scores in $(\mu + \phi_d, \bar{\theta}]$ arise with positive probability.
- iv. The maximal revenue guarantee is at most $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}})$, and this bound is attained when the support of F is binary, that is, $\{\underline{\theta}, \bar{\theta}\}$.

Proposition 2 partially characterizes the robustly optimal test-fee structures and provides a tight upper bound on the maximal revenue guarantee.²¹ There is always a robust solution in the step-exponential-step class, the disclosure fee is strictly positive in any robust solution, and the testing fee is strictly positive if and only if the robustly optimal score distribution has a step at the top.

In the robustly optimal test in the step-exponential-step class, the score τ_0 can be interpreted as a “failing” score and the scores in $[\tau_1, \tau_3]$ can be interpreted as “passing” scores. Even if the support of the distribution of asset values is finite, the test uses a continuum of scores. Because the passing scores are distributed exponentially with a potential step at the top, the robustly

21. Proposition 1 already established that the maximal revenue guarantee is bounded away from full surplus, but the bound provided was not tight.

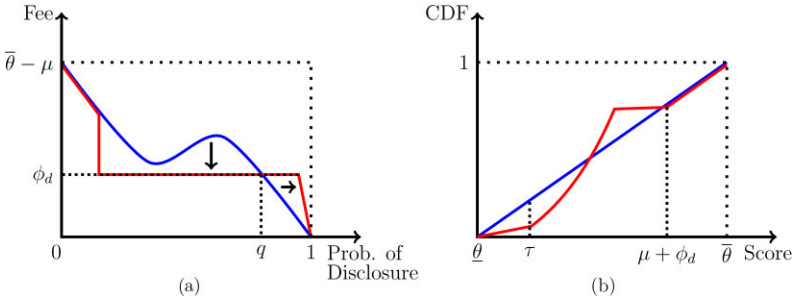


FIGURE VII

How an Intermediary Gains from “Flattening” the Robust Demand Curve

Panel (a) depicts a demand curve in dark gray (blue; color version online), and by flattening the demand curve the intermediary can induce a higher probability of disclosure at disclosure fee ϕ_d . Panel (b) shows that this can be done without changing the score distribution above $\mu + \phi_d$, which guarantees that (P) continues to be satisfied without changing the testing fee.

optimal test assigns higher passing scores more frequently than lower passing scores.

We show after the proof of Proposition 2(i) that if the intermediary uses this test, and charges the testing fee in equation (7) and a disclosure fee slightly below those in equation (6), then this test-fee structure has a unique equilibrium. In this equilibrium, the asset is tested with probability 1 and the agent discloses his score whenever it exceeds τ_1 . If the agent does not disclose a score, he obtains a market price of τ_0 . Therefore, the distribution of market prices involves an atom at τ_0 , an exponential distribution on $[\tau_1, \tau_2]$, and potentially an atom at τ_3 .

We can use the robust demand curve approach described in Section II to obtain an intuition for Proposition 2(i). Consider a test-fee structure with disclosure fee ϕ_d in which the probability of disclosure in an adversarial equilibrium is q . Suppose, as is shown in Figure VII, Panel (a), that the demand correspondence is not flat to the left of the point (q, ϕ_d) . Then we can modify the test-fee structure and improve the intermediary’s robust revenue as follows: flatten the demand correspondence to the left of (q, ϕ_d) and push it to the right of (q, ϕ_d) by changing the test score distribution. Doing this increases the disclosure probability in the adversarial equilibrium for any disclosure fee slightly lower than ϕ_d . Moreover, as seen in Figure VII, Panel (b), this modification can be done without changing the score distribution on scores

above $\mu + \phi_d$, so the option value of a test, given by the right side of (P), remains unchanged. Therefore, this modification does not generate a new equilibrium in which the agent doesn't pay the testing fee. Thus, the intermediary can increase the probability of disclosure by decreasing the disclosure fee arbitrarily slightly and without changing the testing fee, which improves her revenue guarantee.²²

A similar logic shows that in fact every robustly optimal test-fee structure has a score distribution that is exponential over a nondegenerate interval (Lemma 11 in the Appendix). Outside of that interval, there is some flexibility because scores below τ_1 are those that the agent strictly prefers not to disclose, and scores above $\mu + \phi_d$, which exceeds τ_2 , are those for which (HE) is slack. In the step-exponential-step distribution scores below τ_1 are pooled as a single score and scores above $\mu + \phi_d$ are pooled as a single score. This creates a score distribution that is a mean-preserving contraction of the distribution F of the asset value. But there may be other robustly optimal distributions that are the same on the interval $[\tau_1, \tau_2]$ and are also mean-preserving contractions of F .

Turning to disclosure fees, we prove Proposition 2(ii) by contradiction. We show that if the intermediary did not charge a disclosure fee, then the maximal revenue guarantee would come from charging only a testing fee and using a binary test. But we can improve on this by using a test-fee structure with a step-exponential distribution and a strictly positive disclosure fee.

Proposition 2(iii) follows from the participation constraint (P). If scores in $(\mu + \phi_d, \bar{\theta}]$ arise with positive probability, then even if the market expects the asset to not be tested, the agent strictly prefers to have the asset tested at a testing fee of 0 and a disclosure fee of ϕ_d . Increasing the testing fee slightly while keeping the disclosure fee unchanged increases the intermediary's revenue in the adversarial equilibrium and does not generate a new equilibrium in which the asset is not tested. We use this logic in

22. A reader may wonder how this is possible while the total surplus is constant and (w-P) binds. This is because (w-P) is a nonstandard participation constraint. The modification to the test decreases the agent's payoff in the adversarial equilibrium without affecting his indifference between paying the testing fee and not paying it if the market expects him not to pay it, which is what (w-P) represents.

Proposition 4 to provide conditions on primitives that guarantee a strictly positive testing fee. Building on this logic, Proposition 9 in the [Online Appendix](#) shows that extracting revenue with a testing fee is in fact the only reason that scores above $\mu + \phi_d$ may exist. Namely, if the intermediary is restricted to only use disclosure fees (i.e., testing must be free), there exists a test-fee structure in the step-exponential-step class that is robustly optimal and in which there is no score above $\mu + \phi_d$.

We prove Proposition 2(iv) by considering a relaxed problem in which the score distribution need not be a mean-preserving contraction of distribution F , while still having the same expectation, that is, $E_G[s] = E_F[\theta]$. We solve this relaxed problem completely, and its value provides an upper bound on the maximal revenue guarantee. This bound is tight when the support of F is binary because the mean-preserving contraction condition is then equivalent to $E_G[s] = E_F[\theta]$. The following result describes the complete solution for this case.

PROPOSITION 3. Suppose that the support of distribution F is $\{\underline{\theta}, \bar{\theta}\}$.

The unique robustly optimal test-fee structure includes:

- i. No testing fee but a strictly positive disclosure fee: $\phi_t^* = 0$ and $\phi_d^* = \bar{\theta} - \mu$.
- ii. The following marginal score distribution that has an atom at $\underline{\theta}$, a gap above it, and then an exponential form (with no atom at the top):

$$(8) \quad G^*(s) = \begin{cases} e^{\frac{\underline{\theta}-\mu}{\bar{\theta}-\mu}} & \text{if } s \in [\underline{\theta}, \underline{\theta} + \bar{\theta} - \mu) \\ e^{\frac{s-\bar{\theta}}{\bar{\theta}-\mu}} & \text{if } s \in [\underline{\theta} + \bar{\theta} - \mu, \bar{\theta}]. \end{cases}$$

The resulting revenue guarantee is $R_M = (\bar{\theta} - \mu)(1 - e^{\frac{\underline{\theta}-\mu}{\bar{\theta}-\mu}})$.

Proposition 3 shows that for binary asset values $\{\underline{\theta}, \bar{\theta}\}$, there is a unique robustly optimal test-fee structure. The robustly optimal test has an atom at the lowest possible score $\underline{\theta}$ and an exponential distribution on the interval $[\underline{\theta} + \bar{\theta} - \mu, \bar{\theta}]$. The disclosure fee is the only source of revenue. In the adversarial equilibrium, the asset is always tested and the agent pays the disclosure fee whenever the realized score is in $(\underline{\theta} + \bar{\theta} - \mu, \bar{\theta}]$.

The test and equilibrium can be interpreted as follows. The score $\underline{\theta}$ is a “failing” score and the scores in the interval

$(\underline{\theta} + \bar{\theta} - \mu, \bar{\theta}]$ are “passing” scores. If the asset value is high, the test always generates a passing score. But if the asset value is low, the test generates a passing score with an interior probability. The test is “noisy” in that passing scores partially pool low and high asset values.²³ By partially pooling low and high asset values, the intermediary obtains revenue not only when the asset value is high but also with positive probability when the asset value is low.

Let us compare this maximal revenue guarantee with the maximal revenue the intermediary could obtain if she could select not only the test-fee structure but also the equilibrium. Recall from Section III.B that if the intermediary could select the equilibrium, then she could extract the full informational surplus $R_F = \mu - \underline{\theta}$ by using a fully revealing test and charging a high testing fee. We consider the ratio $\frac{R_M}{R_F}$, where R_M is the revenue guarantee from Proposition 3. Holding $\underline{\theta}$ and $\bar{\theta}$ fixed in the binary case, the ratio $\frac{R_M}{R_F}$ is decreasing in the expected value μ of the asset. In other words, the impact of adversarial equilibrium selection on the intermediary’s revenue increases with the probability that the asset value is high.

We know from Proposition 2 that the robustly optimal disclosure fee is always strictly positive; Proposition 3 shows that the robustly optimal testing fee may be 0. We now provide a sufficient condition on the distribution F for the optimal testing fee to be strictly positive.

PROPOSITION 4. If the distribution F of the asset’s value is log-concave and has a strictly positive density, then any robustly optimal test-fee structure includes a strictly positive testing fee.

Several commonly studied distributions, such as the uniform distribution over $[\underline{\theta}, \bar{\theta}]$ and the truncated normal and Pareto distributions, are log-concave (Bagnoli and Bergstrom 2005). Log-concavity requires the tail of the distribution to be less heavy than the tail of the exponential distribution.

23. If the asset value is low, the agent obtains a passing score with probability $1 - \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \mu} e^{\frac{\bar{\theta} - \mu}{\bar{\theta} - \mu}}$. Holding the high and low values fixed, increasing the probability of the high value (i.e., a first-order stochastically dominating shift of the value distribution F) increases the conditional probability that a low-value asset obtains a passing score. Holding the ex ante probability of the high value fixed, this conditional probability does not vary with the low and high values.

VI. EXTENSIONS

This section considers three extensions. [Section VI.A](#) describes a connection between charging only testing fees and binary certification. [Section VI.B](#) shows that our results also hold when testing is costly for the intermediary. A special case is when the set of tests available to the intermediary is restricted. [Section VI.C](#) shows that the intermediary does not gain from being able to offer the agent multiple pieces of evidence.

VI.A. Binary Certification and Testing Fees

A commonly observed form of certification is when the intermediary uses a test with only two possible scores (e.g., pass/fail). We show that such binary certification is robustly optimal if the intermediary can charge only a testing fee. We also show that if the intermediary is restricted to using binary certification, then charging only a testing fee is sufficient. Thus, our analysis draws a connection between binary certification and charging only upfront fees.

To see this, suppose first that the intermediary can charge only a testing fee. We say that a test G is binary if its support comprises two (distinct) scores.

PROPOSITION 5. If the intermediary is restricted to charging only a testing fee, there exists a robustly optimal test-fee structure that includes:

- i. A binary test with a high score $s_H \equiv E[\theta | \theta \geq \mu]$ and a low score $s_L \equiv E[\theta | \theta < \mu]$.
- ii. A strictly positive testing fee that makes the agent's participation constraint ([w-P](#)) bind.

[Proposition 5](#) shows that if the intermediary can charge only a testing fee, she can achieve her maximal revenue guarantee with a binary test. Moreover, the robustly optimal binary test, which is unique up to specifying which score to assign to an asset value that is exactly equal to μ , has an appealing structure. It distinguishes above-average asset values from those that are below average: the high score s_H pools together all above-average asset values, and the low score s_L pools together all below-average asset values.

To see why Proposition 5 holds, it is helpful to contrast the setting here with our general model. In the general model, the intermediary's revenue guarantee comes from the agent paying the testing fee and, with some probability, the disclosure fee. For a fixed positive disclosure fee, changing the test to maximize the option value enables the intermediary to charge a higher testing fee. But changing the test may also introduce equilibria in which the agent discloses the score with low probability, so the intermediary's revenue guarantee may in fact decrease. In the setting of Proposition 5, the intermediary cannot charge a disclosure fee. Therefore, the option value of a test is the intermediary's only source of revenue, so she chooses a test that maximizes this option value. The binary test described above achieves this.²⁴

In the opposite direction, suppose that the intermediary can use only binary tests.

PROPOSITION 6. If the intermediary is restricted to using only binary tests, there exists a robustly optimal test-fee structure that includes:

- i. The binary test described in Proposition 5.
- ii. No disclosure fee and a strictly positive testing fee that makes the agent's participation constraint ($w-P$) bind.

The argument underlying Proposition 6 is that once restricted to binary tests, the intermediary cannot guarantee herself a revenue higher than the option value of the test. The "above / below average" binary test identified in Proposition 5 maximizes this option value. She can then extract the entire option value by charging the testing fee that makes the participation constraint ($w-P$) bind. She does not gain from using a disclosure fee because any increase in revenue from the disclosure fee is offset by a reduction in the testing fee required to satisfy ($w-P$).

Propositions 5 and 6 connect two features that appear in some markets for hard information: the use of testing fees (without disclosure fees) and binary certification. Our analysis shows that if an intermediary is restricted to choosing a test-fee structure that satisfies one of these two features, then it is robustly optimal for her to choose a test-free structure that satisfies the other feature.

24. In fact, any test that assigns scores weakly higher than μ to asset values higher than μ and scores lower than μ to asset values lower than μ maximizes the option value. Thus, a fully revealing test is also robustly optimal when the intermediary can charge only testing fees.

This also shows that the value of charging disclosure fees and using richer tests, as in Proposition 2, comes from the intermediary's ability to do both simultaneously. Relatedly, we see from this analysis that regulations that restrict the intermediary to charging only a testing fee or using only binary tests can improve the agent's welfare. But these results also imply that even with these restrictions, the intermediary can still extract surplus from the agent.

VI.B. Costly Tests

Our analysis assumed that testing is costless for the intermediary, but in reality, evaluating the asset may be costly for her. Suppose that the intermediary incurs a cost of $c(G)$ when conducting a test whose marginal score distribution is G . We assume that c is monotone in the Blackwell order, that is, garbling a test weakly reduces its cost; in other words, whenever G' is a mean-preserving contraction of G , $c(G') \leq c(G)$.²⁵ This condition is standard when information acquisition is costly and corresponds to a less informative test being less costly.²⁶ A special case is when the intermediary starts with a finite set of initial tests and can costlessly garble those tests to obtain additional tests.

The following is a corollary of our existing results.

PROPOSITION 7. With costly testing, there exists a robustly optimal test-fee structure in the step-exponential-step class, and every robustly optimal test-fee structure uses a strictly positive disclosure fee.

Proposition 7 is a corollary of Lemma 11 in the Appendix, which is one of the steps in the proof of Proposition 2. Lemma 11 shows that for any test-fee structure (G, ϕ) , there exists a test-fee structure in the step-exponential-step class that uses a mean-preserving contraction of G and has a weakly higher revenue guarantee. Because such a test-fee structure is a garbling of G , it has a weakly lower cost. Therefore, there exist a robustly optimal test-fee structure in the step-exponential-step class.²⁷ The argument

25. To guarantee that a solution to the relevant maximization problem exists, we assume that c is lower semi-continuous with respect to the weak* topology.

26. Two recent studies that assume monotonicity in the Blackwell order are De Oliveira et al. (2017) and Pomatto, Strack, and Tamuz (2020).

27. Moreover, if costs are strictly monotone in the Blackwell order, this argument implies that every robustly optimal test-fee structure is in the step-exponential-step class.

used to prove Proposition 2 proves that the optimal disclosure fee is strictly positive.

VI.C. Multiple Pieces of Evidence

Our analysis assumed that the intermediary provided the agent with a single piece of evidence that he could verifiably disclose. One could envision the intermediary providing the agent with multiple pieces of evidence and a choice of which to disclose. We show that this additional generality does not change the robustly optimal test-fee structures.

To see this, suppose that the intermediary designs, along with the test-fee structure, an arbitrary evidence structure, which specifies a message space \mathcal{M} and the set of messages available for each score, described by $M : S \rightrightarrows \mathcal{M}$. We call this an evidence-test-fee structure. Our baseline model corresponds to $M(s) = \{s\}$ for every score s . The agent first decides whether to have the asset tested and pay the testing fee ϕ_t . If he pays the testing fee, he observes the test score s . Then, if the agent pays the disclosure fee ϕ_d , he can disclose a message m in $M(s)$ to the buyers. If the agent does not have the asset tested or does not disclose any message, the buyers observe the null message N . The following result shows that using evidence-test-fee structures does not improve the maximal revenue guarantee.

PROPOSITION 8. For any evidence-test-fee structure and any adversarial equilibrium of the induced game, there exists a test-fee structure that has an adversarial equilibrium with the same revenue for the intermediary.

If the intermediary chose the equilibrium in addition to the test-fee structure, Proposition 8 would follow from the logic of the revelation principle. But adversarial equilibrium selection introduces a new consideration: because in the game induced by an evidence-test-fee structure the agent has more actions than in the game induced by a test-fee structure, it also has more deviations, and these deviations may exclude strategy profiles that would otherwise be equilibria. Thus, it is conceivable that under adversarial equilibrium selection, some evidence-test-fee structure generates a higher revenue for the intermediary than any test-fee structure. We show this is not the case. Consider an adversarial equilibrium of a game induced by an evidence-test-fee structure with fees ϕ . This equilibrium generates a distribution \tilde{G} of

equilibrium prices paid to the agent. The equilibrium also includes a critical price \tilde{p} such that the agent is indifferent between disclosing any evidence that leads to price \tilde{p} and nondisclosure. We show that $(\tilde{G}, \phi, \tilde{p})$ satisfies constraints (P) and (HE). Therefore the test-fee structure (\tilde{G}, ϕ) has a highest equilibrium threshold of \tilde{p} and an identical revenue guarantee to that of the evidence-test-fee structure.

VII. CONCLUSION

Sellers of financial or physical assets often disclose to buyers hard information purchased from an information intermediary, such as a credit rating agency or an appraiser, about the value of the assets. One rationale for the existence of such information intermediaries is that their presence generates economic value by, for example, mitigating moral hazard or facilitating assortative matching. A less obvious rationale for the presence of such intermediaries, even if they provide no economic value, is that once sellers of assets have the option to acquire hard information from an intermediary, potential buyers may have an unfavorable view of assets whose sellers do not disclose favorable information.

Our article investigates the scope of this second rationale. In a setting in which information has no social value, we study how an intermediary designs and prices evidence for a disclosure game between an asset owner and the asset market. Our main finding is that even if the equilibria of the disclosure game are chosen to minimize the intermediary's revenue, she can still guarantee herself a high revenue across equilibria. We study how she accomplishes this.

First, she uses option value as a carrot. The agent prefers not to have the asset tested and for the market to believe that the asset has not been tested. To prevent the agent from credibly refraining from testing, the intermediary chooses a test-fee structure that is irresistible because of its option value: the test has sufficiently high scores and the fees are sufficiently low that even if the market were to believe that the agent has refrained from testing, he cannot resist having the asset tested and disclosing high scores. Thus, in equilibrium, the market correctly expects the agent to have the asset tested and treats nondisclosure with prejudice. In this way, the intermediary guarantees herself revenue by exploiting the agent's inability to commit.

Second, the intermediary optimally uses positive disclosure fees and noisy tests. To develop an intuition for the optimal combination of test and fees, we propose a demand curve approach in which every test corresponds to a robust demand curve, and the intermediary can be thought of as choosing an optimal price on an optimal robust demand curve. The optimal disclosure fee maximizes the area of a rectangle under the robust demand curve, so the optimal robust demand curve has a rectangular component, which leads to an exponential distribution of scores.

Our analysis may shed light on some strategic forces in the context of credit ratings. In the credit rating industry, it is common for a rating agency to offer an informal “shadow rating” to an issuer of a financial instrument and charge the issuer only if he would like to complete the rating process so the rating can be disclosed to potential investors. Our work illustrates that a credit rating agency may use shadow ratings and disclosure fees to make it difficult for issuers to credibly refrain from obtaining a rating. The step-exponential-step distribution identified in Proposition 2 shows that the credit rating agency should (i) use a large number of scores even if the number of possible asset values is small, (ii) assign higher scores more frequently than lower ones, and (iii) generate bunching at the lowest score and sometimes at the highest score. These normative implications are consistent with a concern that credit rating agencies may distort rating schemes to maximize revenue. Finally, parts ii and iii of Proposition 2 shows that a credit rating agency should always charge a disclosure fee but only charge a testing fee if the agency chooses to have bunching at the highest score.

As caveats to our analysis, let us highlight the role of a few important assumptions. First, we have focused on two-part tariffs, which are often used in practice. It would be interesting to study a broader range of pricing structures, including disclosure fees that depend on the realized test score and fees paid by prospective buyers of the asset. Second, we have assumed that the intermediary can commit to a rating scheme (in line with recent work on information design and certification). This is often rationalized on the basis of dynamic or reputational incentives, but it may be interesting to investigate a model with weaker forms of commitment.²⁸ Third, we have focused on a monopolistic

28. In the context of credit rating agencies, Mathis, McAndrews, and Rochet (2009), Bolton, Freixas, and Shapiro (2012), and Frenkel (2015) consider

intermediary. With multiple intermediaries, the asset owner may be able to “shop around” and disclose his favorite rating. The ability to shop around may benefit the asset owner by allowing him to keep more of the surplus. However, this ability could also lead the market to draw inferences from the number of scores disclosed, which in turn could generate a “rat race” that compels the asset owner to disclose multiple scores. This effect would complement the negative inferences the market draws from nondisclosure in our model with a single intermediary. By increasing pressure on the asset owner to disclose hard information, the presence of competing intermediaries may end up hurting the asset owner. Identifying the conditions under which one effect dominates the other may be an interesting direction for future research.

APPENDIX

A.1. Proof of Lemma 1

Proof. Consider a test-fee structure (G, ϕ) that satisfies (P). Assume toward a contradiction that there exists an equilibrium in which the agent has the asset tested with probability strictly less than 1. In this equilibrium, let p_N be the price that the agent obtains when he does not disclose his score. The agent discloses his score if $s - \phi_d > p_N$ and does not disclose his score if $s - \phi_d < p_N$. Therefore, the expected value conditional on nondisclosure is no higher than the prior expected value: $p_N \leq \mu$.

Because the agent has the asset tested with probability strictly less than 1, his equilibrium payoff is p_N . Consider the following deviation: having the asset tested with probability 1 and disclosing a score of s if and only if $s > p_N + \phi_d$. The payoff from this deviation is $-\phi_t + \int_{p_N + \phi_d}^{\bar{\theta}} (s - \phi_d) dG + \int_{\bar{\theta}}^{p_N + \phi_d} p_N dG$, where the first term is the testing fee, the second is the payoff from disclosing scores higher than $p_N + \phi_d$, and the third is the payoff from concealing scores lower than $p_N + \phi_d$. This deviation is profitable

reputational models as underlying credit rating agencies' ability to commit. In the context of information design, [Mathevet, Pearce, and Stachetti \(2019\)](#) and [Best and Quigley \(2020\)](#) study when long-run incentives enable a designer to commit to an information structure.

because

$$\begin{aligned}
 & -\phi_t + \int_{p_N + \phi_d}^{\bar{\theta}} (s - \phi_d) dG + \int_{\underline{\theta}}^{p_N + \phi_d} p_N dG - p_N \\
 & = -\phi_t + \int_{p_N + \phi_d}^{\bar{\theta}} (s - p_N - \phi_d) dG \\
 & \geq -\phi_t + \int_{\mu + \phi_d}^{\bar{\theta}} (s - \mu - \phi_d) dG \\
 & > 0,
 \end{aligned}$$

where the first inequality follows from $p_N \leq \mu$ and the second inequality follows from (P). Therefore, the agent has the asset tested with probability 1 in every equilibrium.

Now consider a test-fee structure (G, ϕ) that violates (P), so

$$(9) \quad \phi_t \geq \int_{\mu + \phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG.$$

Suppose the agent has the asset tested with probability 0 (and off-path discloses scores weakly greater than $\mu + \phi_d$). The price following nondisclosure is μ . This constitutes an equilibrium. First, the agent's behavior is sequentially rational. Second, by deviating and taking the test, the agent's expected payoff is no higher than μ :

$$\begin{aligned}
 & \int_{\underline{\theta}}^{\mu + \phi_d} \mu dG + \int_{\mu + \phi_d}^{\bar{\theta}} [s - \phi_d] dG - \phi_t \\
 & = \mu + \int_{\mu + \phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG - \phi_t \leq \mu,
 \end{aligned}$$

where the inequality follows from equation (9). \square

A.2. Proof of Lemma 2

Proof. We first show that the intermediary can obtain positive revenue robustly by using nonnegative fees. To see this, consider a test-fee structure (G', ϕ') , where $G' = F$, $\phi'_d = 0$, and $\phi'_t > 0$. Because F is nondegenerate, we have $F(\mu) < 1$. So $G'(\mu + \phi'_d) < 1$ and the right side of (P), $\int_{\mu + \phi'_d}^{\bar{\theta}} [s - (\mu + \phi'_d)] dG'$, is strictly positive.

Thus, for sufficiently small $\phi'_t > 0$ the test-fee structure (G', ϕ') satisfies (P). By Lemma 1, $R(G', \phi') = \phi'_t > 0$.

Now consider any test-fee structure (G, ϕ) . If $R(G, \phi) \leq 0$, then, as argued already, there exists (G', ϕ') with nonnegative fees such that $R(G, \phi) < R(G', \phi')$, and the lemma follows. So suppose for the rest of the proof that $R(G, \phi) > 0$, which by Lemma 1 implies that (G, ϕ) satisfies (P) and the agent has the asset tested with probability 1.

If $\phi_d < 0$, then it is straightforward to show that in every equilibrium of (G, ϕ) , all scores disclose, so $0 < R(G, \phi) = \phi_t + \phi_d$. Set $G' = G$, $\phi'_d = 0$, and $\phi'_t = \phi_t + \phi_d + \varepsilon$. Integration by parts shows that the right side of (P) plus ϕ'_d ,

$$\int_{\mu+\phi'_d}^{\bar{\theta}} [s - (\mu + \phi'_d)] dG' + \phi'_d = - \int_{\mu+\phi'_d}^{\bar{\theta}} G(s) ds + \bar{\theta} - \mu,$$

is increasing in ϕ'_d . Therefore, because $\phi'_d > \phi_d$, we have $\phi_t + \phi_d < \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG + \phi_d \leq \int_{\mu+\phi'_d}^{\bar{\theta}} [s - (\mu + \phi'_d)] dG'$. So for small enough $\varepsilon > 0$, (G', ϕ') satisfies (P) and $R(G', \phi') = \phi'_t > R(G, \phi)$.

Now suppose $\phi_t < 0$ and $\phi_d \geq 0$. If $\int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG = 0$, then $G(\mu + \phi_d) = 1$, which implies that $\tau = \mu + \phi_d$ is the threshold satisfying (HE), and therefore $R(G, \phi) = \phi_t + 0 < 0$. So it must be that $\int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG > 0$, but then the test-fee structure (G', ϕ') , where $G' = G$, $\phi'_d = \phi_d$, and $\phi'_t = 0 > \phi_t$, satisfies (P). Because the disclosure decision does not depend on the testing fee (because ϕ_t does not appear in (HE)), the test-fee structure (G', ϕ') has the same highest equilibrium threshold as does (G, ϕ) , and so $R(G', \phi') > R(G, \phi)$. \square

A.3. Proof of Lemma 3

The following lemma is used in our proof.

LEMMA 5. Suppose f is an increasing function defined on $[a, b] \subset \mathbb{R}$, and g is a continuous function defined on $[a, b]$. If $f(a) \geq g(a)$ and $f(b) \leq g(b)$, there exists $x^* \in [a, b]$ such that $f(x^*) = g(x^*)$. Moreover, $\bar{x} = \max\{x \mid f(x) = g(x)\}$ exists and $\bar{x} = \sup\{x \in [a, b] \mid f(x) \geq g(x)\}$.

Proof. Let $S \equiv \{x \in [a, b] \mid f(x) \geq g(x)\}$ and $\bar{x} \equiv \sup S$ (the supremum is well defined because S is nonempty and bounded above by b). Consider an increasing sequence $x_n \in S$ that converges to \bar{x} .

Since f is an increasing function, $f(\bar{x}) \geq \lim_{n \rightarrow \infty} f(x_n)$. Because $x_n \in S$, we have $f(x_n) \geq g(x_n)$, so

$$f(\bar{x}) \geq \lim_{n \rightarrow \infty} f(x_n) \geq \lim_{n \rightarrow \infty} g(x_n) = g(\bar{x}),$$

where the equality holds because g is continuous.

Now we prove that $f(\bar{x}) = g(\bar{x})$. Suppose toward a contradiction that $f(\bar{x}) > g(\bar{x})$, so $\bar{x} < b$. Because g is continuous, $f(\bar{x}) > g(\bar{x} + \varepsilon)$ for some $\varepsilon \in (0, b - \bar{x})$. Because f is increasing,

$$f(\bar{x} + \varepsilon) \geq f(\bar{x}) > g(\bar{x} + \varepsilon),$$

which contradicts $\bar{x} = \sup S$. Moreover, since by definition $f(x) < g(x)$ for all $x \notin S$, it follows that $\bar{x} = \max\{x \mid f(x) = g(x)\}$. \square

Proof of Lemma 3(i). A threshold τ is an equilibrium threshold if and only if

$$E_G[s \mid s \leq \tau] = \tau - \phi_d.$$

Define $a(\tau) = E_G[s \mid s \leq \tau]$ and $b(\tau) = \tau - \phi_d$. Since $a(\tau)$ is bounded above by μ and $b(\tau) \rightarrow \infty$ as $\tau \rightarrow \infty$, there exists $\bar{\tau}$ large enough such that $a(\tau) < b(\tau)$ for all $\tau \geq \bar{\tau}$. So any equilibrium threshold τ must be in $[s_G, \bar{\tau}]$.

We have $a(s_G) = s_G \geq b(s_G)$ and $a(\bar{\tau}) \leq b(\bar{\tau})$, a is increasing, and b is continuous. Therefore, Lemma 5 shows that the set of equilibrium thresholds is nonempty and a highest equilibrium threshold exists. \square

Proof of Lemma 3(ii). The “if” part is immediate from the definition of the highest equilibrium threshold. For the “only if” part, suppose that τ is the highest equilibrium threshold of the test-free structure (G, ϕ) , so $\tau - \phi_d = E[s \mid s \leq \tau]$ and for all $\tau' > \tau$, $\tau' - \phi_d \neq E[s \mid s \leq \tau']$. We show that in fact $\tau' - \phi_d > E[s \mid s \leq \tau']$ for all $\tau' > \tau$.

Suppose toward a contradiction that for some $\tau' > \tau$ we have $\tau' - \phi_d < E[s \mid s \leq \tau']$. Define $a, b, \bar{\tau}$ as in the proof of Lemma 3(i). Since $a(\tau') > b(\tau')$, $a(\bar{\tau}) \leq b(\bar{\tau})$, a is increasing, and b is continuous, Lemma 5 shows that some τ^* in $(\tau', \bar{\tau}) \subset (\tau, \bar{\tau})$ is an equilibrium threshold, a contradiction. \square

Proof of Lemma 3(iii). We show that the agent disclosing a score if and only if the score exceeds the highest threshold τ is an adversarial equilibrium. Suppose there exists another

equilibrium that gives the intermediary a lower revenue. In this equilibrium, let $\tilde{\tau} = p_N + \phi_d$, where p_N is the price following nondisclosure. All scores strictly greater than $\tilde{\tau}$ disclose and all scores strictly lower than $\tilde{\tau}$ conceal, so the equilibrium price satisfies

$$E[s < \tilde{\tau}] \leq p_N \leq E[s | s \leq \tilde{\tau}].$$

The intermediary's revenue from this equilibrium is $\phi_t + \phi_d[1 - G(\tilde{\tau}) + \lambda(G(\tilde{\tau}) - \sup_{s < \tilde{\tau}} G(s))]$, where $\lambda \in [0, 1]$ is the probability that the agent discloses when $s = \tilde{\tau}$. Since this equilibrium gives the intermediary a lower revenue,

$$1 - G(\tilde{\tau}) + \lambda \left(G(\tilde{\tau}) - \sup_{s < \tilde{\tau}} G(s) \right) < 1 - G(\tau),$$

which implies $\tilde{\tau} > \tau$. But then from part ii, the characterization of τ , and the fact that $\tilde{\tau} > \tau$, we have

$$\tilde{\tau} - \phi_d > E[s | s \leq \tilde{\tau}] \geq p_N,$$

which contradicts $\tilde{\tau} = p_N + \phi_d$. □

A.4. Proof of Lemma 4

We prove Lemma 4 by first proving part iii, then part i, and finally part ii. To prove part iii we use the following lemma.

LEMMA 6. If $(G^n, \tau^n) \rightarrow (G, \tau)$, and $\lim_{n \rightarrow \infty} G^n(\tau^n)$ exists, then $\lim_{n \rightarrow \infty} G^n(\tau^n) \leq G(\tau)$.

Proof. It suffices to show that for any $c > G(\tau)$, there exists N such that for $n \geq N$, $G^n(\tau^n) < c$. Because G is right continuous, it follows that for any $c > G(\tau)$, there exists $\bar{\varepsilon} > 0$ such that

$$(10) \quad c > G(\tau + 2\bar{\varepsilon}) + \bar{\varepsilon}.$$

Because $\tau^n \rightarrow \tau$, there exists N_1 such that for $n > N_1$, $\tau^n \leq \tau + \bar{\varepsilon}$ and hence

$$(11) \quad G^n(\tau^n) \leq G^n(\tau + \bar{\varepsilon}).$$

The topology of weak convergence is metrized by the Levy metric L , which for distribution G and any distribution H assigns

distance

$$L(G, H) = \inf\{\varepsilon > 0 | H(x - \varepsilon) - \varepsilon \leq G(x) \leq H(x + \varepsilon) + \varepsilon \text{ for all } x\}.$$

Thus, if G^n converges weakly to G , there exists N_2 such that for all $n > N_2$, $L(G, G^n) < \bar{\varepsilon}$ and hence

$$(12) \quad G(\tau + 2\bar{\varepsilon}) + \bar{\varepsilon} \geq G^n(\tau + \bar{\varepsilon}).$$

Combining equations (10), (11), and (12), it follows that for every $n \geq \max\{N_1, N_2\}$,

$$c > G(\tau + 2\bar{\varepsilon}) + \bar{\varepsilon} \geq G^n(\tau + \bar{\varepsilon}) \geq G^n(\tau^n).$$

□

Proof of Lemma 4(iii). Let us begin by verifying (w-P). We have

$$\begin{aligned} \phi_t &= \lim_{n \rightarrow \infty} \phi_t^n \leq \lim_{n \rightarrow \infty} \int_{\mu + \phi_d^n}^{\bar{\theta}} [s - (\mu + \phi_d^n)] dG^n(s) \\ &= \int_{\mu + \phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG(s), \end{aligned}$$

where the equalities follow from taking limits and the inequality follows from (G^n, ϕ^n, τ^n) satisfying (P). So (G, ϕ) satisfies (w-P).

We now turn to (w-HE). Since $\tau^n - \phi_d^n = E[s | s \leq \tau^n] \leq \mu$ and $\phi_d^n \rightarrow \phi_d$, τ^n is uniformly bounded from above. Also $\tau^n \geq \underline{\theta}$ so τ^n is bounded from below. By the Bolzano-Weierstrass theorem, there exists a subsequence n_k and τ such that $\lim_{k \rightarrow \infty} \tau^{n_k} = \tau$.

For any $\tau' > \tau$, we show that $\phi_d G(\tau') \leq \int_{\underline{\theta}}^{\tau'} G(s) ds$, which by integration by parts means that $\tau' - \phi_d \geq E_G[s | s \leq \tau']$. Indeed, if $\phi_d G(\tau') > \int_{\underline{\theta}}^{\tau'} G(s) ds$, there exists a small enough ε such that $\phi_d G(\tau' + \varepsilon) > \int_{\underline{\theta}}^{\tau' + \varepsilon} G(s) ds$ and G is continuous at $\tau' + \varepsilon$. Continuity implies that $\lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau' + \varepsilon) = \phi_d G(\tau' + \varepsilon)$. In addition, $\tau' > \tau$ implies that for large enough k , $\tau' + \varepsilon > \tau' > \tau^{n_k}$. Because $(G^{n_k}, \phi^{n_k}, \tau^{n_k})$ satisfies (HE), we have $\tau' - \phi_d^{n_k} > E_{G^{n_k}}[s | s \leq \tau']$, which means that $\phi_d^{n_k} G^{n_k}(\tau' + \varepsilon) < \int_{\underline{\theta}}^{\tau' + \varepsilon} G^{n_k}(s) ds$. But then we

have a contradiction:

$$\begin{aligned}\phi_d G(\tau' + \varepsilon) &> \int_{\underline{\theta}}^{\tau' + \varepsilon} G(s) ds = \lim_{k \rightarrow \infty} \int_{\underline{\theta}}^{\tau' + \varepsilon} G^{n_k}(s) ds \\ &\geq \lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau' + \varepsilon) = \phi_d G(\tau' + \varepsilon).\end{aligned}$$

Next we show that $\phi_d G(\tau) = \int_{\underline{\theta}}^{\tau} G(s) ds$, which by integration by parts means that $\tau - \phi_d = E_G[s | s \leq \tau]$. Since G is right continuous and for all $\tau' > \tau$, $\phi_d G(\tau') \leq \int_{\underline{\theta}}^{\tau'} G(s) ds$, we must have $\phi_d G(\tau) \leq \int_{\underline{\theta}}^{\tau} G(s) ds$. Because $(G^{n_k}, \phi^{n_k}, \tau^{n_k})$ satisfies (HE), we have $\tau^{n_k} - \phi_d^{n_k} = E_{G^{n_k}}[s | s \leq \tau^{n_k}]$, and from integration by parts, $\phi_d^{n_k} G^{n_k}(\tau^{n_k}) = \int_{\underline{\theta}}^{\tau^{n_k}} G^{n_k}(s) ds$. So $\lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau^{n_k})$ exists and equals $\int_{\underline{\theta}}^{\tau} G(s) ds$. Therefore,

$$\phi_d G(\tau) \geq \lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau^{n_k}) = \int_{\underline{\theta}}^{\tau} G(s) ds,$$

where the inequality holds from Lemma 6. Therefore, $\phi_d G(\tau) = \int_{\underline{\theta}}^{\tau} G(s) ds$. Also since $\lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau^{n_k})$ is between $\phi_d G(\tau)$ and $\int_{\underline{\theta}}^{\tau} G(s) ds$, we have $\phi_d G(\tau) = \lim_{k \rightarrow \infty} \phi_d^{n_k} G^{n_k}(\tau^{n_k})$.

Now that we have shown that (G, ϕ, τ) satisfies (w-P) and (w-HE), we turn to revenue:

$$\begin{aligned}\lim_{n \rightarrow \infty} \phi_t^n + \phi_d^n (1 - G^n(\tau^n)) &= \phi_t + \lim_{k \rightarrow \infty} \phi_d^{n_k} (1 - G^{n_k}(\tau^{n_k})) \\ &= \phi_t + \phi_d (1 - G(\tau)).\end{aligned}$$

So (G, ϕ, τ) generates revenue that is the limit of the revenues generated by $(G^n, (\phi_t^n, \phi_d^n))$ and τ^n . \square

We use the following lemma to prove Lemma 4(i) and Lemma 4(ii).

LEMMA 7. For every test-fee structure and threshold (G, ϕ, τ) that satisfy (w-P) and (w-HE), there exists a sequence of test-fee structures and thresholds $\{(G^n, \phi^n, \tau^n)\}_{n=1, 2, \dots}$ such that (i) for each n , (G^n, ϕ^n, τ^n) satisfy (P) and (HE), (ii) (G^n, ϕ^n) converges to (G, ϕ) , and (iii) $\hat{R}(G, \phi, \tau) \leq \lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n)$.

Proof. Consider a test-fee structure and threshold (G, ϕ, τ) that satisfy (w-P) and (w-HE), so

$$\begin{aligned}\phi_t &\leq \int_{\mu+\phi_d}^{\bar{\theta}} (s - \mu - \phi_d) dG(s) \\ \tau - \phi_d &= E[s|s \leq \tau] \\ \tau' - \phi_d &\geq E[s|s \leq \tau'] \quad \text{for all } \tau' > \tau.\end{aligned}$$

Letting $\phi_d^n = \phi_d - \frac{1}{n}$ and $\phi_t^n = \phi_t - \frac{1}{n}$, we have

$$\phi_t^n < \phi_t \leq \int_{\mu+\phi_d}^{\bar{\theta}} (s - \mu - \phi_d) dG(s) \leq \int_{\mu+\phi_d^n}^{\bar{\theta}} (s - \mu - \phi_d^n) dG(s),$$

so (P) is satisfied.

From Lemma 3, we know that under (G, ϕ^n) , the highest equilibrium threshold $\hat{\tau}^n$ exists. Moreover,

$$\begin{aligned}\tau - \phi_d^n &> \tau - \phi_d = E[s|s \leq \tau] \\ \tau' - \phi_d^n &> \tau' - \phi_d \geq E[s|s \leq \tau'] \quad \text{for all } \tau' > \tau.\end{aligned}$$

This implies that the highest threshold $\hat{\tau}_n$ satisfies $\hat{\tau}_n < \tau$. So the revenue under (G, ϕ^n) is

$$\phi_t^n + \phi_d^n(1 - G(\hat{\tau}_n)) \geq \phi_t^n + \phi_d^n(1 - G(\tau)) \geq \phi_t + \phi_d(1 - G(\tau)) - \frac{2}{n},$$

where the last term goes to $\hat{R}(G, \phi, \tau)$ as $n \rightarrow \infty$. Notice that $G(\hat{\tau}^n)$ is bounded so there exists a subsequence n_k so that $G(\hat{\tau}^{n_k})$ converges. Taking such a subsequence and letting $k \rightarrow \infty$, we have

$$\begin{aligned}\lim_{k \rightarrow \infty} \hat{R}(G^{n_k}, \phi^{n_k}, \tau^{n_k}) &= \lim_{k \rightarrow \infty} \phi_t^{n_k} + \phi_d^{n_k}(1 - G(\hat{\tau}_{n_k})) \geq \phi_t + \phi_d(1 - G(\tau)) \\ &= \hat{R}(G, \phi, \tau).\end{aligned}$$

□

Proof of Lemma 4(i). Consider a sequence $(G^n, (\phi_t^n, \phi_d^n))$ and τ^n satisfying (P) and (HE) and generating value V^n that converges to R_M . By Lemma 2 we can assume that the fees (ϕ_t^n, ϕ_d^n) are nonnegative. This implies that $\phi_t^n \leq \int_{\mu}^{\bar{\theta}} (s - \mu) dG^n(s) \leq \bar{\theta} - \mu$. Also $\phi_d^n \leq \bar{\theta} - \mu$, since otherwise (P) implies that $\phi_t^n < 0$.

Now we have a sequence $(G^n, (\phi_t^n, \phi_d^n))$ such that $\phi_t^n \in [0, \bar{\theta} - \mu]$, $\phi_d^n \in [0, \bar{\theta} - \mu]$, and $G^n \in \Gamma(F)$. Proposition 1 of Kleiner, Moldovanu, and Strack (2021) proves that $\Gamma(F)$ is compact, which implies that there exists a converging subsequence $(G^{n_k}, (\phi_t^{n_k}, \phi_d^{n_k})) \rightarrow (G, (\phi_t, \phi_d))$. From Lemma 4(iii), we can find a τ such that $(G, (\phi_t, \phi_d), \tau)$ satisfies (w-P) and (w-HE) and generates revenue R_M . Moreover, by Lemma 7, any test-fee structure and threshold satisfying (w-P) and (w-HE) must generate a revenue of at most R_M , so $(G, (\phi_t, \phi_d), \tau)$ is an optimal solution to the relaxed problem. \square

Proof of Lemma 4(ii). By Lemma 7, there exists a sequence of test-fee structures and thresholds $\{(G^n, \phi^n, \tau^n)\}_{n=1,2,\dots}$ such that (i) for each n , (G^n, ϕ^n, τ^n) satisfies (P) and (HE), (ii) (G^n, ϕ^n) converges to (G, ϕ) , and (iii) $\lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n) \geq \hat{R}(G, \phi, \tau) = R_M$. By definition of R_M , $\lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n) \leq R_M$, so $\lim_{n \rightarrow \infty} \hat{R}(G^n, \phi^n, \tau^n) = R_M$. \square

A.5. Proof of Proposition 2

We prove Proposition 2 in reverse order, beginning with iv and ending with i. We then prove, as discussed in the text following Proposition 2, that slightly lowering the disclosure fee of a robustly optimal step-exponential-step test-fee structure results in a test-fee structure that has a unique equilibrium.

Proof of Proposition 2(iv). We consider a relaxed problem in which the mean-preserving contraction constraints are relaxed to requiring the score distribution to have same expectation as the prior mean:

$$(RE) \quad \max_{(G, \phi, \tau) \in \Delta[\underline{\theta}, \bar{\theta}] \times \mathbb{R}^3} \hat{R}(G, \phi, \tau)$$

$$\text{s.t. (w-P), (w-HE), and } E_G[s] = E_F[s].$$

Recall that $\hat{R}(G, \phi, \tau) \equiv \phi_t + \phi_d(1 - G(\tau))$ is the revenue of a test-fee structure (G, ϕ) and a threshold τ . This is a relaxed problem because $G \in \Gamma(F)$ implies that $G \in \Delta[\underline{\theta}, \bar{\theta}]$ and $E_G[s] = E_F[s]$.

We use two lemmas to solve the relaxed problem. The first lemma shows that in any optimal test-fee structure, the only possible score above $\mu + \phi_d$ is $\bar{\theta}$. In other words, the score distribution G is flat from $\mu + \phi_d$ to $\bar{\theta}$, with possibly a discrete jump at $\bar{\theta}$.

LEMMA 8. If a test-fee structure (G, ϕ) with a weak-highest equilibrium threshold τ is an optimal solution to the relaxed problem (RE), then $G(s) = G(\mu + \phi_d)$ for all $s \in [\mu + \phi_d, \bar{\theta}]$.

Proof. Using integration by parts, the testing fee of an optimal test-fee structure is

$$\phi_t = \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG(s) = - \int_{\mu+\phi_d}^{\bar{\theta}} G(s) ds + (\bar{\theta} - (\mu + \phi_d)).$$

Thus, the revenue function is

$$\hat{R}(G, \phi, \tau) = -\phi_d G(\tau) - \int_{\mu+\phi_d}^{\bar{\theta}} G(s) ds + (\bar{\theta} - \mu).$$

The weak-highest equilibrium threshold τ satisfies $\phi_d G(\tau) = \int_{\underline{\theta}}^{\tau} G(s) ds$. To see this, note that if $G(\tau) = 0$, then the equality holds trivially. If $G(\tau) > 0$, then from the equality in (w-HE) and integration by parts,

$$\phi_d = \tau - E_G[s | s \leq \tau] = \frac{\tau G(\tau) - \int_{\underline{\theta}}^{\tau} s dG(s)}{G(\tau)} = \frac{\int_{\underline{\theta}}^{\tau} G(s) ds}{G(\tau)},$$

which means that $\phi_d G(\tau) = \int_{\underline{\theta}}^{\tau} G(s) ds$. Substituting this equality into the expression for the revenue, we have

$$\begin{aligned} (13) \quad \hat{R}(G, \phi, \tau) &= - \int_{\underline{\theta}}^{\tau} G(s) ds - \int_{\mu+\phi_d}^{\bar{\theta}} G(s) ds + (\bar{\theta} - \mu) \\ &= \int_{\tau}^{\mu+\phi_d} G(s) ds, \end{aligned}$$

where the second equality followed from the constraint that the integral of G is $\bar{\theta} - \mu$.

Now consider an optimal test-fee structure (G, ϕ) with a weak-highest equilibrium threshold τ , and suppose for contradiction that $G(s) > G(\mu + \phi_d)$ for some $s \in [\mu + \phi_d, \bar{\theta}]$. Construct a distribution G' as follows. Let $G'(s) = \alpha G(s)$ for some α and all $s \leq \mu + \phi_d$, and $G'(s) = G'(\mu + \phi_d)$ for all $s \in [\mu + \phi_d, \bar{\theta}]$. By the assumption that $G(s) > G(\mu + \phi_d)$ for some $s \in [\mu + \phi_d, \bar{\theta}]$, there exists an $\alpha > 1$ such that the areas under G and G' are equal. Define $\phi'_d = \phi_d$, and ϕ'_t so that the upper bound on the testing fee (w-P) holds with equality for distribution G' .

We now show that τ is a weak-highest equilibrium threshold of the test-fee structure (G', ϕ') and gives higher revenue than (G, ϕ) . Indeed, for any τ' such that $\tau \leq \tau' \leq \mu + \phi_d$, because G' is equal to G multiplied by α , we have

$$\frac{\int_{\underline{\theta}}^{\tau'} G(s) ds}{G(\tau')} = \frac{\int_{\underline{\theta}}^{\tau} G(s) ds}{G(\tau)}.$$

Thus, from integration by parts, $\tau' - E_{G'}[s|s \leq \tau'] = \tau' - E_G[s|s \leq \tau'] \geq \phi_d$, with equality at $\tau' = \tau$. In addition, for all $\tau' \geq \mu + \phi_d$, since G' is flat we have

$$\frac{\int_{\underline{\theta}}^{\tau'} G'(s) ds}{G'(\tau')} \geq \frac{\int_{\underline{\theta}}^{\mu + \phi_d} G(s) ds}{G(\mu + \phi_d)} \geq \phi_d.$$

It remains to show that the revenue of (G', ϕ') is higher than that of (G, ϕ) . This fact follows directly from equation (13), since $G' > G$ below $\mu + \phi_d$.²⁹ \square

The second lemma that we use to solve the relaxed problem (RE) bounds the rate at which the integral of a score distribution can grow given the (w-HE) constraint.

LEMMA 9. Suppose that $\phi_d > 0$. Let τ be a weak-highest equilibrium threshold. Then for any τ_a and τ_b with $\tau \leq \tau_a \leq \tau_b$,

$$(14) \quad \int_{\underline{\theta}}^{\tau_b} G(s) ds \leq e^{\frac{1}{\phi_d}(\tau_b - \tau_a)} \int_{\underline{\theta}}^{\tau_a} G(s) ds,$$

with equality if and only if (w-HE) holds with equality for all thresholds in $[\tau_a, \tau_b]$.

Proof. Using integration by parts, (w-HE) can be written as

$$\phi_d \leq \tau' - E[s|s \leq \tau'] = \frac{\int_{\underline{\theta}}^{\tau'} G(s) ds}{G(\tau')}$$

for all $\tau' \geq \tau$, with equality at $\tau' = \tau$. The right-hand side is the inverse of the derivative of $\ln(\int_{\underline{\theta}}^{\tau'} G(s) ds)$ with respect to τ' . Thus,

29. If $\tau = \mu + \phi_d$, $\hat{R}(G, \phi, \tau) = 0$. As we have shown in the proof of Lemma 2, the optimal revenue is strictly positive. So in an optimal test-fee structure (G, ϕ) with threshold τ , $\tau < \mu + \phi_d$.

$\frac{d}{d\tau'}(\ln(\int_{\underline{\theta}}^{\tau'} G(s)ds)) \leq \frac{1}{\phi_d}$. Integrating from τ_a to τ_b we obtain

$$\ln\left(\int_{\underline{\theta}}^{\tau_b} G(s)ds\right) - \ln\left(\int_{\underline{\theta}}^{\tau_a} G(s)ds\right) \leq \frac{1}{\phi_d}(\tau_b - \tau_a).$$

Taking exponents on both sides and rearranging yields equation (14), with equality if and only if (w-HE) holds with equality for all thresholds in $[\tau_a, \tau_b]$. \square

Equipped with Lemma 8 and Lemma 9, we solve the relaxed problem (RE).

LEMMA 10. The value of (RE) is $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}})$, which is achieved by a unique solution $\phi_t^* = 0$, $\phi_d^* = \bar{\theta} - \mu$, and

$$G^*(s) = \begin{cases} e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}} & \text{if } s \in [\underline{\theta}, \underline{\theta} + \bar{\theta} - \mu) \\ e^{\frac{s-\underline{\theta}}{\bar{\theta}-\mu}} & \text{if } s \in [\underline{\theta} + \bar{\theta} - \mu, \bar{\theta}]. \end{cases}$$

Proof. We first argue that the test-fee structure (G^*, ϕ^*) achieves the revenue bound $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}})$. We later show that $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}})$ is an upper bound on revenue and therefore (G^*, ϕ^*) is optimal. We start by verifying that (G^*, ϕ^*) is a feasible solution.

First, we show that (w-P) is satisfied. In fact, it holds with equality: since $\mu + \phi_d^* = \bar{\theta}$, $\int_{\mu+\phi_d^*}^{\bar{\theta}} [s - (\mu + \phi_d^*)] dG^*(s) = 0 = \phi_t^*$.

Second, we show that the constraint $E_{G^*}[s] = E_F[s]$ is satisfied. From the definition of G^* , for any $\tau' \geq \underline{\theta} + \bar{\theta} - \mu$ we have

$$\int_{\underline{\theta}}^{\tau'} G^*(s)ds = (\bar{\theta} - \mu)e^{\frac{\bar{\theta}-\mu}{\bar{\theta}-\mu}} + (\bar{\theta} - \mu)e^{\frac{s-\underline{\theta}}{\bar{\theta}-\mu}} \Big|_{\underline{\theta}+\bar{\theta}-\mu}^{\tau'} = (\bar{\theta} - \mu)e^{\frac{\tau'-\bar{\theta}}{\bar{\theta}-\mu}}.$$

Thus, in particular, $\int_{\underline{\theta}}^{\bar{\theta}} G^*(s)ds = \bar{\theta} - \mu$ and hence, by integration by parts,

$$E_{G^*}[s] = \int_{\underline{\theta}}^{\bar{\theta}} s dG^*(s) = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} G^*(s)ds = \mu = E_F[s].$$

Third, we show that $\tau = \underline{\theta} + \bar{\theta} - \mu$ satisfies (w-HE). For any $\tau' \geq \underline{\theta} + \bar{\theta} - \mu$,

$$\begin{aligned} E[s|s \leq \tau'] &= \frac{\int_{\underline{\theta}}^{\tau'} s dG^*(s)}{G^*(\tau')} = \frac{\tau' G(\tau') - \int_{\underline{\theta}}^{\tau'} G^*(s) ds}{G^*(\tau')} \\ &= \tau' - \frac{(\bar{\theta} - \mu) e^{\frac{\tau' - \bar{\theta}}{\bar{\theta} - \mu}}}{e^{\frac{\tau' - \bar{\theta}}{\bar{\theta} - \mu}}} = \tau' - \phi_d^*. \end{aligned}$$

Therefore, $\tau = \underline{\theta} + \bar{\theta} - \mu$ satisfies (w-HE) and is a weak-highest equilibrium threshold.

This establishes that (G^*, ϕ^*) is a feasible solution. The revenue of the test fee structure (G^*, ϕ^*) with weak-highest equilibrium threshold $\tau = \underline{\theta} + \bar{\theta} - \mu$ is

$$\phi_t^* + \phi_d^*(1 - G(\underline{\theta} + \bar{\theta} - \mu)) = 0 + (\bar{\theta} - \mu) \left(1 - e^{\frac{\bar{\theta} - \mu}{\bar{\theta} - \mu}}\right).$$

We prove that the above is the value of (RE) by showing that the revenue of any test-fee structure is at most $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta} - \mu}{\bar{\theta} - \mu}})$.

Consider a test-fee structure (G, ϕ) with a weak-highest equilibrium threshold τ that is an optimal solution to the relaxed problem (RE). For any τ' such that $\tau \leq \tau' \leq \mu + \phi_d$, the total area under G is

$$\begin{aligned} \bar{\theta} - \mu &= \int_{\underline{\theta}}^{\tau'} G(s) ds + \int_{\tau'}^{\bar{\theta}} G(s) ds \\ (15) \quad &\leq \phi_d e^{\frac{1}{\phi_d}(\tau' - \tau)} G(\tau) + (\bar{\theta} - \tau') G(\mu + \phi_d), \end{aligned}$$

where the inequality follows from Lemma 9 and the fact that $G(s) \leq G(\mu + \phi_d)$ for all $s \leq \mu + \phi_d$, and $G(s) = G(\mu + \phi_d)$ for all $s \in [\mu + \phi_d, \bar{\theta}]$ by Lemma 8. Let $\tau' = \phi_d + \frac{\mu - \bar{\theta}(1 - G(\mu + \phi_d))}{G(\mu + \phi_d)}$. Notice that $\tau' \in [\tau, \mu + \phi_d]$ because from the definition of τ , $\tau = \phi_d + E[s|s \leq \tau] \leq \phi_d + E[s|s \leq \mu + \phi_d] = \tau'$, and $\mu < \bar{\theta}$ implies $\frac{\mu - \bar{\theta}(1 - G(\mu + \phi_d))}{G(\mu + \phi_d)} < \mu$.

Now from equation (15) we have

$$\begin{aligned}
 G(\tau) &\geq \frac{1}{\phi_d} e^{-\frac{1}{\phi_d}(\tau' - \tau)} ((\bar{\theta} - \mu) - (\bar{\theta} - \tau') G(\mu + \phi_d)) \\
 &= \frac{1}{\phi_d} e^{-\frac{1}{\phi_d}(\tau' - \tau)} \left((\bar{\theta} - \mu) - \left(\frac{\bar{\theta} - \mu}{G(\mu + \phi_d)} - \phi_d \right) G(\mu + \phi_d) \right) \\
 &= e^{-\frac{1}{\phi_d}(\tau' - \tau)} G(\mu + \phi_d) \\
 &\geq e^{-\frac{1}{\phi_d}(\bar{\theta} - \underline{\theta} - \frac{\bar{\theta} - \mu}{G(\mu + \phi_d)})} G(\mu + \phi_d),
 \end{aligned}$$

where the last inequality follows from $\underline{\theta} + \phi_d \leq \tau$.

We use the inequality above and Lemma 8 to bound revenue. By Lemma 8, the distribution is flat above $\mu + \phi_d$, so the testing fee is

$$\phi_t = \int_{\mu + \phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG(s) = (1 - G(\mu + \phi_d))(\bar{\theta} - (\mu + \phi_d)).$$

The revenue is therefore

$$\begin{aligned}
 \hat{R}(G, \phi, \tau) &\leq \phi_d \left[1 - e^{-\frac{1}{\phi_d}(\bar{\theta} - \underline{\theta} - \frac{\bar{\theta} - \mu}{G(\mu + \phi_d)})} G(\mu + \phi_d) \right] \\
 &\quad + (1 - G(\mu + \phi_d))(\bar{\theta} - (\mu + \phi_d)).
 \end{aligned}$$

The above expression is increasing in ϕ_d . Since $\phi_d \leq \bar{\theta} - \mu$, an upper bound on the revenue is obtained by substituting $\phi_d = \bar{\theta} - \mu$ into the above expression, which yields

$$(\bar{\theta} - \mu) \left(1 - e^{-\frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \mu} + \frac{1}{G(\mu + \phi_d)}} G(\mu + \phi_d) \right).$$

This expression is increasing in $G(\mu + \phi_d)$. Because $G(\mu + \phi_d) \leq 1$, an upper bound on the revenue is obtained by substituting $G(\mu + \phi_d) = 1$ into the above expression, which yields $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \mu}})$ and completes the proof. \square

Proof of Proposition 2(iv). The solution to the relaxed problem in Lemma 10 is $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \mu}})$. Therefore, $(\bar{\theta} - \mu)(1 - e^{\frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \mu}})$ is an upper bound on the revenue of any test-fee structure. Moreover, if

the support of the prior is binary (i.e., $\{\underline{\theta}, \bar{\theta}\}$), $G \in \Gamma(F)$ is equivalent to $G \in \Delta[\underline{\theta}, \bar{\theta}]$ and $E_G[s] = E_F[s]$, so the bound is attained. \square

Proof of Proposition 2(iii). Proof. In a robustly optimal test-fee structure, the constraint (w-P) must bind, otherwise the intermediary can strictly increase the revenue by increasing ϕ_t , so $\phi_t = \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG$. Now, $\int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG > 0$ if and only if $G(\mu + \phi_d) < 1$, which then implies the result. \square

Proof of Proposition 2(iv). Proof. Suppose toward a contradiction that there exists a robustly optimal test-fee structure (G, ϕ) with $\phi_d = 0$. By (w-P), the revenue guarantee is

$$\phi_t \leq \int_{\mu}^{\bar{\theta}} (s - \mu) dG(s) = (\bar{\theta} - \mu) - \int_{\mu}^{\bar{\theta}} G(s) ds,$$

which is maximized when the inequality holds as an equality and $\int_{\mu}^{\bar{\theta}} G(s) ds$ is minimized. Since $\int_{\mu}^{\bar{\theta}} G(s) ds \geq \int_{\mu}^{\bar{\theta}} F(s) ds$ for any G that is a mean-preserving contraction of F , we have $\int_{\mu}^{\bar{\theta}} G(s) ds = \int_{\mu}^{\bar{\theta}} F(s) ds$. Now consider a test G' with support $\{s_L, s_H\}$ that is obtained from F by pooling all scores strictly below μ into s_L and similarly pooling all scores weakly above μ into s_H . We have $s_L < s_H$ since F is nondegenerate. Since $\int_{\mu}^{\bar{\theta}} G'(s) ds = \int_{\mu}^{\bar{\theta}} F(s) ds$, the test-fee structure (G', ϕ) satisfies (w-P) and so it is also robustly optimal.

The revenue guarantee of (G', ϕ) is the testing fee

$$(16) \quad \int_{\mu}^{\bar{\theta}} (s - \mu) dG'(s) = \frac{(s_H - \mu)(\mu - s_L)}{s_H - s_L}.$$

However, by Lemma 10, there exists a test-fee structure (G'', ϕ') with revenue guarantee $(s_H - \mu)(1 - e^{\frac{s_L - \mu}{s_H - \mu}})$ such that $G'' \in \Delta[s_L, s_H]$ and $E_{G''}[s] = E_{G'}[s]$. Since the support of G' is binary, $E_{G''}[s] = E_{G'}[s]$ implies that G'' is a mean-preserving contraction of G' and therefore of F . Because $\mu \in (s_L, s_H)$, the revenue guarantee of (G'', ϕ') is strictly higher than equation (16), contradicting the optimality of (G', ϕ) . \square

Proof of Proposition 2(i). The following lemma is the main step in the proof of Proposition 2(i). It shows that for any test-fee structure, there exists a test-fee structure that is in the step-exponential-step class and has a weakly higher revenue guarantee. The lemma further establishes that any robustly optimal test must be exponential over an interval.

LEMMA 11. For any test-fee structure (G, ϕ) with $\phi_d > 0$ and a weak-highest threshold τ_1 , there exists a mean-preserving contraction G' of G , a fee structure ϕ' with $0 \leq \phi'_t \leq \phi_t$, and a threshold τ'_1 such that the test-fee structure (G', ϕ') is in the step-exponential-step class, τ'_1 is a weak-highest threshold, and the revenue of (G', ϕ', τ'_1) weakly exceeds the revenue of (G, ϕ, τ_1) . Furthermore, if (G, ϕ) is robustly optimal and has weak-highest equilibrium threshold τ_1 , then $\phi_d > 0$ and there exists a threshold $\tau_2 \in [\tau_1, \mu + \phi_d]$ such that G is exponential from τ_1 to τ_2 and is flat from τ_2 to $\mu + \phi_d$, that is,

$$\begin{aligned} G(s) &= G(\tau_1)e^{\frac{1}{\phi_d}(s-\tau_1)} \text{ for all } s \in [\tau_1, \tau_2], \\ G(s) &= G(\mu + \phi_d) \text{ for all } s \in [\tau_2, \mu + \phi_d]. \end{aligned}$$

Proof. Consider a test-fee structure (G, ϕ) with a weak-highest threshold τ_1 . By definition of τ_1 we have $\tau_1 = E_G[s|s \leq \tau_1] + \phi_d \leq \mu + \phi_d$. Suppose that $\tau_1 = \mu + \phi_d$. Then $E_G[s|s \leq \tau_1] = \mu$ so $G(\tau_1) = 1$. The intermediary's revenue from the disclosure fee is $\phi_d(1 - G(\tau_1)) = 0$. Moreover, from (w-P), the revenue from the testing fee is at most $\int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)]dG = 0$ since $G(\mu + \phi_d) = 1$. So the intermediary receives zero revenue, which is never optimal. We thus assume that $\tau_1 < \mu + \phi_d$.

We first consider the case $G(\mu + \phi_d) = G(\tau_1)$. Let $\tau_a = E_G[s|s \leq \tau_1]$ and $\tau_b = E_G[s|s > \mu + \phi_d]$. From the definition of τ_1 , we have $\tau_a = E_G[s|s \leq \tau_1] = \tau_1 - \phi_d$. The mean constraint requires that $G(\tau_1)\tau_a + (1 - G(\tau_1))\tau_b = \mu$, which implies that $G(\tau_1) = \frac{\tau_b - \mu}{\tau_b - \tau_a}$.

Given the test G , and to satisfy (w-P), the highest testing fee the intermediary can charge is $\phi_t = \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)]dG(s)$. The revenue from the disclosure fee under equilibrium threshold τ_1 is $\phi_d(1 - G(\tau_1))$. So the intermediary's robust revenue under test G

is at most

$$\begin{aligned}
 & \int_{\mu+\phi_d}^{\bar{\theta}} [s - (\mu + \phi_d)] dG(s) + \phi_d(1 - G(\tau_1)) \\
 &= \bar{\theta} - \mu - \phi_d - \int_{\mu+\phi_d}^{\bar{\theta}} G(s) ds + \phi_d - \phi_d G(\tau_1) \\
 &= \int_{\underline{\theta}}^{\mu+\phi_d} G(s) ds - \int_{\underline{\theta}}^{\tau_1} G(s) ds \\
 &= \int_{\tau_1}^{\mu+\phi_d} G(s) ds \\
 &= G(\tau_1)(\mu + \phi_d - \tau_1) \\
 &= \frac{(\tau_b - \mu)(\mu - \tau_a)}{\tau_b - \tau_a}.
 \end{aligned}$$

We now construct the following distribution G' in the step-exponential-step class with a null last step:

$$G'(s) = \begin{cases} 0 & \text{if } s \in [\underline{\theta}, \tau_a) \\ e^{\frac{\tau_a - \mu}{\tau_b - \mu}} & \text{if } s \in [\tau_a, \tau_a + \tau_b - \mu) \\ e^{\frac{s - \tau_b}{\tau_b - \mu}} & \text{if } s \in [\tau_a + \tau_b - \mu, \tau_b] \\ 1 & \text{if } s \in [\tau_b, \bar{\theta}]. \end{cases}$$

Observe that G' is a mean-preserving contraction of G , and $\tau'_1 = \tau_a + \tau_b - \mu$ is a weak-highest equilibrium threshold for the test-fee structure (G', ϕ') with $\phi'_d = \tau_b - \mu$ and $\phi'_t = 0 \leq \phi_t$. In this equilibrium, the intermediary's revenue is $(\tau_b - \mu)(1 - e^{\frac{\tau_a - \mu}{\tau_b - \mu}}) > \frac{(\tau_b - \mu)(\mu - \tau_a)}{\tau_b - \tau_a}$.

Now consider the case $G(\mu + \phi_d) > G(\tau_1)$. We construct a class of distributions G_α parametrized by $\alpha \in [0, 1]$:

$$(17) \quad G_\alpha(s) = \begin{cases} \alpha G(s) & \text{if } s \leq \tau_1 \\ \min\{\alpha G(\tau_1) e^{\frac{1}{\phi_d}(s - \tau_1)}, G(\mu + \phi_d)\} & \text{if } \tau_1 < s < \mu + \phi_d \\ G(s) & \text{if } s \geq \mu + \phi_d. \end{cases}$$

Notice that G_α is well defined since $\phi_d > 0$, and is increasing and between 0 and 1. Therefore, G_α is a distribution.

We first show that there exists an $\alpha \leq 1$ such that the integrals of G_α and G are equal. We do this with a continuity argument. Let $\tau_2(\alpha)$ be the lowest score $\tau' > \tau_1$ such that $G_\alpha(\tau') = G_\alpha(\mu + \phi_d)$. Consider $\alpha = 1$. We have

$$\begin{aligned} \int_{\underline{\theta}}^{\tau_2(\alpha)} G_1(s) ds &= e^{\frac{1}{\phi_d}(\tau_2(\alpha) - \tau_1)} \int_{\underline{\theta}}^{\tau_1} G_1(s) ds = e^{\frac{1}{\phi_d}(\tau_2(\alpha) - \tau_1)} \int_{\underline{\theta}}^{\tau_1} G(s) ds \\ (18) \quad &\geq \int_{\underline{\theta}}^{\tau_2(\alpha)} G(s) ds, \end{aligned}$$

where the inequality follows from Lemma 9. As a result, because G_1 is weakly higher than G for all scores above $\tau_2(\alpha)$, we have $\int_{\underline{\theta}}^{\bar{\theta}} G_1(s) ds \geq \int_{\underline{\theta}}^{\bar{\theta}} G(s) ds$. Further,

$$\int_{\underline{\theta}}^{\mu + \phi_d} G_0(s) ds = 0 \leq \int_{\underline{\theta}}^{\mu + \phi_d} G(s) ds.$$

As a result, since G_0 and G are equal above $\mu + \phi_d$, we have $\int_{\underline{\theta}}^{\bar{\theta}} G_0(s) ds \leq \int_{\underline{\theta}}^{\bar{\theta}} G(s) ds$. The integral of G_α increases continuously in α . Therefore, there exists some $\alpha \leq 1$ such that the integrals of G_α and G are equal. For the rest of the proof, fix such an α .

We now show that G_α is a mean-preserving contraction of G . Since $\alpha \leq 1$, the integral of G_α up to any threshold $\tau' \leq \tau_1$ is weakly lower than that of G . For $\tau' \geq \tau_2(\alpha)$, since the overall integrals of G and G_α are equal and G_α is weakly higher than G above $\mu + \phi_d$, we have $\int_{\underline{\theta}}^{\tau'} G_\alpha(s) ds \leq \int_{\underline{\theta}}^{\tau'} G(s) ds$. Finally, for any τ' such that $\tau_1 \leq \tau' \leq \tau_2(\alpha)$ we have

$$\begin{aligned} \int_{\underline{\theta}}^{\tau'} G_\alpha(s) ds &= e^{-\frac{1}{\phi_d}(\tau_2(\alpha) - \tau')} \int_{\underline{\theta}}^{\tau_2(\alpha)} G_\alpha(s) ds \leq e^{-\frac{1}{\phi_d}(\tau_2(\alpha) - \tau')} \int_{\underline{\theta}}^{\tau_2(\alpha)} G(s) ds \\ &\leq \int_{\underline{\theta}}^{\tau'} G(s) ds, \end{aligned}$$

where the second inequality follows from Lemma 9. Thus, G_α is a mean-preserving contraction of G .

Test-fee structure (G_α, ϕ) has a weakly higher revenue guarantee than (G, ϕ) . Because the two distributions are equal above $\mu + \phi_d$, the (w-P) constraint is satisfied for (G_α, ϕ) . Furthermore, τ_1 is a weak-highest equilibrium threshold for (G_α, ϕ) . Since $\alpha \leq 1$, we

have $G_\alpha(\tau_1) \geq G(\tau_1)$, which implies that $\hat{R}(G_\alpha, \phi, \tau_1) \geq \hat{R}(G, \phi, \tau_1)$. In fact, if $\alpha < 1$, then $\hat{R}(G_\alpha, \phi, \tau_1) > \hat{R}(G, \phi, \tau_1)$.

To see the first statement of the lemma, consider a score distribution G' that is equal to G_α except that it pools the scores below τ_1 and also pools the scores above τ_2 . Formally,

$$G'(s) = \begin{cases} G_\alpha(\tau_1) & \text{if } s \in [\tau_0, \tau_1], \\ G_\alpha(s) & \text{if } s \in [\tau_1, \tau_2], \\ 1 & \text{if } s \in [\tau_3, \theta], \end{cases}$$

where τ_0 and τ_3 are such that the integrals of G' and G_α are equal. Notice that the test-fee structure (G', ϕ) is in the step-exponential-step class, with a nondegenerate exponential part because $\tau_2 > \tau_1$. Distribution G' is a mean-preserving contraction of G_α and therefore of G . Finally, the robust revenue of the test-fee structure (G', ϕ) with weak-highest equilibrium threshold τ_1 is equal to that of (G_α, ϕ) , and therefore is at least that of (G, ϕ) . This establishes the first statement of the lemma.

To see the second statement, suppose that (G, ϕ) is optimal. Then $\phi_d > 0$ by Proposition 2(ii). Recall that if $\alpha < 1$, the robust revenue of (G_α, ϕ) is strictly higher than that of (G, ϕ) . Therefore, $\alpha = 1$. Now let $\tau_2 = \tau_2(1)$. By definition, G is flat from τ_2 to $\mu + \phi_d$. Further, equation (18) holds with equality for $\alpha = 1$, that is, $e^{\frac{1}{\phi_d}(\tau_2 - \tau_1)} \int_{\underline{\theta}}^{\tau_1} G(s) ds = \int_{\underline{\theta}}^{\tau_2} G(s) ds$. Then Lemma 9 implies that (w-HE) holds with equality for all thresholds in $[\tau_1, \tau_2]$. Thus, for any $\tau' \in [\tau_1, \tau_2]$,

$$\left(\frac{d}{d\tau'} \left(\ln \left(\int_{\underline{\theta}}^{\tau'} G(s) ds \right) \right) \right)^{-1} = \frac{\int_{\underline{\theta}}^{\tau'} G(s) ds}{G(\tau')} = \tau' - E[s | s \leq \tau'] = \phi_d.$$

The solution to this differential equation is $G(s) = G(\tau_1) e^{\frac{1}{\phi_d}(s - \tau_1)}$ for all $s \in [\tau_1, \tau_2]$. \square

Proof of Proposition 2(i). Consider any robustly optimal test-fee structure (G, ϕ) . By Proposition 2(ii), $\phi_d > 0$. By Lemma 11, the robust revenue of (G, ϕ) is at most the robust revenue of some step-exponential-step test-fee structure (G', ϕ) where G' is a mean-preserving contraction of G . This (G', ϕ) is therefore robustly optimal. \square

1. *Uniqueness of Equilibrium with Slightly Lower Disclosure Fee.* As mentioned in the discussion that followed Proposition 2, we show in addition that there is a unique equilibrium under any robustly optimal step-exponential-step test-fee structure with a slightly lower disclosure fee. Let (G, ϕ) be a robustly optimal step-exponential-step test-fee structure:

$$G(s) = \begin{cases} \kappa & \text{if } s \in [\tau_0, \tau_1) \\ \kappa e^{\frac{s-\tau_1}{\tau_1-\tau_0}} & \text{if } s \in [\tau_1, \tau_2) \\ 1 & \text{if } s \geq \tau_3, \end{cases}$$

the disclosure fee is $\phi_d = \tau_1 - \tau_0$, and the testing fee is $\phi_t = (1 - \kappa e^{\frac{\tau_2-\tau_1}{\tau_1-\tau_0}})(\tau_3 - (\mu + \phi_d))$. Notice that for any $\tau \in [\tau_1, \tau_2]$,

$$\begin{aligned} E_G[s|s \leq \tau] &= \tau - \frac{\int_{\theta}^{\tau} G(s)ds}{G(\tau)} = \tau - \frac{\kappa(\tau_1 - \tau_0)e^{\frac{\tau-\tau_1}{\tau_1-\tau_0}}}{\kappa e^{\frac{\tau-\tau_1}{\tau_1-\tau_0}}} = \tau - \tau_1 - \tau_0 \\ &= \tau - \phi_d. \end{aligned}$$

For $\tau > \tau_2$, $E_G[s|s \leq \tau] \leq \tau - \phi_d$. For $\tau \in [\tau_0, \tau_1)$, $E_G[s|s \leq \tau] = \tau_0 < \tau - \phi_d$.

Consider a test-fee structure (G, ϕ') with $\phi'_t = \phi_t$ and $\phi'_d = \phi_d - \varepsilon$ for $\varepsilon < \tau_1 - \tau_0$. Then there exists a unique threshold $\tau' = \tau_1 - \varepsilon$ satisfying (HE). Moreover, since (G, ϕ) satisfies (w-P), (G, ϕ') satisfies (P) because $\int_{\mu+\phi_d}^{\theta} [s - (\mu + \phi_d)]dG < \int_{\mu+\phi_d-\varepsilon}^{\theta} [s - (\mu + \phi_d - \varepsilon)]dG$. Therefore, in the unique equilibrium under (G, ϕ') , the asset is tested with probability 1 and the agent discloses all the scores above τ_1 .

A.6. Proof of Proposition 3

Proof. With binary support, the mean-preserving contraction constraints become $E_G[s] = E_F[\theta]$. Therefore, the proposition follows from Lemma 10. \square

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SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online.

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